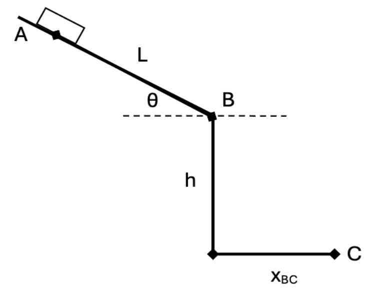
Ramp-Projectile Optimization Problem

Problem Statement:

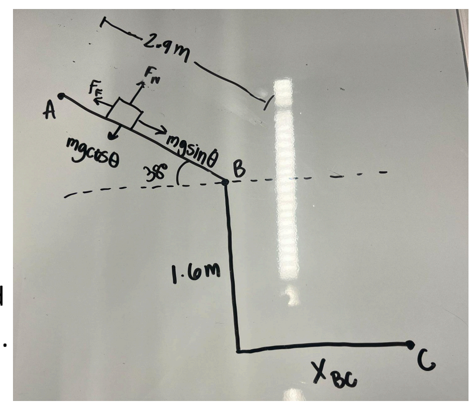


It is given that there is a 72g puck that is sliding down a ramp. This ramp is on top of a counter, so the puck will fall and land a certain amount away from the base of the counter, represented by X_{BC}. The length of the ramp, L, is equal to 2.9 meters, and the angle that the ramp makes with the countertop, θ, is 38°. There is friction present when the puck slides down the ramp, with the coefficient of friction, μ, equalling 0.17. The height of the counter that the ramp is on top of, h, is 1.6 m. The objectives of this problem are to:

a.) Calculate how far the puck lands away from X_{BC}, the base of the counter, after falling from the ramp.

b.) Calculating the angle of the ramp, θ, that would maximize the distance of how far away the puck will land, after falling from the ramp.

Process:



1.) The first thing we did was make an annotated/free body diagram using the information given to us in the problem. This is seen to the right.

2.) Then, we calculated F_N by using the $F_N = mg\cos\theta$ equation. We got $F_N = (0.072)(9.8)\cos(38)$, which came out to be 0.556.

3.) Then, we used the equation $F_F = \mu F_N$ to find what F_F would be. We got $F_F = (0.17)(0.556)$. This came out to be 0.9452.

4.) Then, by looking at our free body diagram, we can derive the equation $ma= mg\sin\theta - F_F$. We chose to use this equation because it'll allow us to solve for a, as we already know all the other values. We can rearrange this equation in terms of a. This would be $a=(mg\sin\theta - F_F)/m$. We used this to solve for a, which came out to be 4.720 m/s^2.

5.) Next, in order to find the final velocity of the ramp portion of the problem, which ends up being the initial velocity of the projectile portion of the problem, we need to use the "no t" big 4 equation. This equation is $v^2 = v_0^2 + 2a\Delta x$. We plugged in the known values and solved for v, which was 5.233 m/s.

6.) The V_x, which is $v*\cos(\theta)$, was multiplied by the time that could be solved by using the no v equation: $\Delta y=v_0$^2-.5gt^2. We could solve for t by substituting Δy with 1.6m and v_0 with 5.233*sin(38), which is the y part of the velocity. Then, we can solve it using a calculator, which we got as 0.3305 seconds.

7.) After this, we used the Google Sheets application to predict the value of X_{BC} for values of θ from 0° to 90°. In order to do this, we would need to repeat our process for each angle, so we instead wrote formulas for each Force as well as for X_{BC}, which we solved using the equation $\Delta x=vt$.

Solution:

We used google sheets to predict the X_{BC} for angles 0° to 90° to confirm our answer. When θ was 38º, the values of each force matched up with our calculations, and we found that the distance was about 1.36m. We also estimated based on results from sheets that 27º would be closest to the optimal angle for this problem. The distance when θ was 27º, the distance was about 1.52m. Note that acceleration, velocity, Vy and Vx are for degree values 0° to 9° are 0 because mgsin$\theta \not> F_F$ this means that the puck is not sliding down the ramp yet, which does not comply with our problem.

A	B	C	D	E	F	G	H	I	J	K
Degrees	Radians	Normal Force	Friction Force	Acceleration	Velocity	V_y	V_x	Determinat (Time Formula)	Time	X_{BC}
0	0	0.7056	0.119952	0	0	0	0	5.6	0.5714285714	0
1	0.01745329252	0.7054925337	0.1199337307	0	0	0	0	5.6	0.5714285714	0
2	0.03490658504	0.7051701675	0.1198789285	0	0	0	0	5.6	0.5714285714	0
3	0.05235987756	0.7046329997	0.11978761	0	0	0	0	5.6	0.5714285714	0
4	0.06981317008	0.7038811939	0.119659803	0	0	0	0	5.6	0.5714285714	0
5	0.0872664626	0.702914979	0.1194955464	0	0	0	0	5.6	0.5714285714	0
6	0.1047197551	0.7017346494	0.1192948904	0	0	0	0	5.6	0.5714285714	0
7	0.1221730476	0.7003405646	0.119057896	0	0	0	0	5.6	0.5714285714	0
8	0.1396263402	0.6987331493	0.1187846354	0	0	0	0	5.6	0.5714285714	0
9	0.1570796327	0.6969128931	0.1184751918	0	0	0	0	5.6	0.5714285714	0
10	0.1745329252	0.6948803505	0.1181296596	0.06106242462	0.5951151677	-0.1033406644	0.586074031	5.600953427	0.5609808942	0.328776334
11	0.1919862177	0.6926361406	0.1177481439	0.2345372671	1.166325919	-0.2225454769	1.144897227	5.604420263	0.5491708966	0.6287442365
12	0.2094395102	0.6901809471	0.117330761	0.4079406672	1.538198904	-0.319809535	1.504585568	5.609124543	0.5397260212	0.8120639821
13	0.2268928028	0.6875155177	0.116877638	0.5812198046	1.83604871	-0.4130210932	1.788990901	5.615210274	0.5308356307	0.9496601132
14	0.2443460953	0.6846406645	0.116388913	0.7543218969	2.091666083	-0.5060198237	2.029534661	5.62281567	0.5221220252	1.059664747
15	0.2617993878	0.681557263	0.1158647347	0.9271942154	2.318992551	-0.6001994375	2.239974796	5.632072386	0.5134564233	1.150129447
16	0.2792526803	0.6782662527	0.115305263	1.099784102	2.525618298	-0.6961547496	2.427780129	5.643104769	0.5047908183	1.225521118
17	0.2967059728	0.6747686358	0.1147106681	1.272038983	2.7162154	-0.794144527	2.597529705	5.656029131	0.4961106739	1.288662213
18	0.3141592654	0.6710654779	0.1140811312	1.443906389	2.893899973	-0.8942642717	2.752262427	5.670953058	0.4874172231	1.341500109
19	0.3316125579	0.6671579069	0.1134168442	1.615333967	3.060871936	-0.9965224272	2.894111273	5.687974767	0.4787196265	1.385467868
20	0.3490658504	0.6630471132	0.1127180092	1.786269498	3.218751791	-1.100877949	3.024637306	5.707182515	0.4700310782	1.421673534
21	0.3665191429	0.6587343489	0.1119848393	1.956660915	3.368773264	-1.207260367	3.145020781	5.728654082	0.4613667056	1.451007877
22	0.3839724354	0.6542209278	0.1112175577	2.126456314	3.511900713	-1.315581163	3.25617764	5.752456327	0.4527423637	1.474209562
23	0.401425728	0.6495082246	0.1104163982	2.295603973	3.648904362	-1.425740519	3.358834175	5.778644826	0.4441739089	1.491906505
24	0.4188790205	0.6445976749	0.1095816047	2.46405237	3.780410526	-1.537631487	3.453576864	5.807263606	0.4356767469	1.504643133
25	0.436332313	0.6394907745	0.1087134317	2.631750192	3.906936282	-1.65114262	3.540886776	5.838344967	0.4272655456	1.51289892
26	0.4537856055	0.6341890791	0.1078121434	2.798646357	4.028914106	-1.766159697	3.621164011	5.871909406	0.418954052	1.517101335
27	0.471238898	0.6286942035	0.1068780146	2.964690028	4.1467098	-1.882566854	3.694745485	5.907965636	0.4107549778	1.5176351
28	0.4886921906	0.6230078215	0.1059113297	3.129830626	4.260635824	-2.000247359	3.761918145	5.946510699	0.4026799327	1.514848946
29	0.5061454831	0.6171316654	0.1049123831	3.294017846	4.370961394	-2.119084134	3.822928974	5.987530172	0.3947393917	1.509060658
30	0.5235987756	0.6110675249	0.1038814792	3.457201677	4.477920246	-2.238960123	3.877992689	6.030998461	0.3869426875	1.500560913
31	0.5410520681	0.6048172474	0.1028189321	3.619332411	4.581716707	-2.359758553	3.927297742	6.076879168	0.379298022	1.489616266
32	0.5585053606	0.5983827366	0.1017250652	3.780360661	4.682530495	-2.481363115	3.971011071	6.125125542	0.3718124926	1.476471525
33	0.5759586532	0.5917659527	0.100600212	3.940237377	4.780520556	-2.603658102	4.00928189	6.175680976	0.3644921299	1.461351695
34	0.5934119457	0.5849689112	0.0994447149	4.098913858	4.875828173	-2.726528513	4.042244753	6.228479568	0.3573419444	1.4444636
35	0.6108652382	0.5779936825	0.09825892602	4.25634177	4.968579502	-2.849860125	4.070022056	6.283446724	0.3503659795	1.425997264
36	0.6283185307	0.5708423912	0.09704320651	4.41247316	5.058887657	-2.973539558	4.092726087	6.340499783	0.3435673699	1.406127138
37	0.6457718232	0.5635172159	0.0957979267	4.567260467	5.146854448	-3.097454329	4.110460727	6.399548681	0.3369484033	1.385013179
38	0.6632251158	0.5560203877	0.09452346592	4.720656543	5.232571829	-3.221492892	4.12332287	6.46049661	0.3305105835	1.362801848
39	0.6806784083	0.5483541904	0.09322021237	4.872614661	5.31612312	-3.345544681	4.131403614	6.523240699	0.3242546957	1.339627021
40	0.6981317008	0.5405209591	0.09188856304	5.023088533	5.397584042	-3.469500144	4.134789262	6.587672673	0.3181808703	1.315610846
41	0.7155849933	0.5325230798	0.09052892357	5.172032323	5.477023596	-3.593250782	4.133562179	6.653679522	0.3122886469	1.29086454
42	0.7330382858	0.5243629889	0.08914170811	5.319400663	5.554504825	-3.716689181	4.127801518	6.721144134	0.306577036	1.265489155
43	0.7504915784	0.5160431719	0.08772733922	5.465148662	5.630085456	-3.839709048	4.117583838	6.789945918	0.3010445786	1.239576292
44	0.7679448709	0.5075661631	0.08628624773	5.609231923	5.703818471	-3.962205245	4.10298364	6.859961399	0.2956894035	1.213208785
45	0.7853981634	0.4989345448	0.08481887262	5.751606558	5.775752595	-4.084073826	4.084073826	6.931064782	0.2905092812	1.186461352
46	0.8028514559	0.4901509462	0.08332566085	5.892229198	5.845932718	-4.205212075	4.060926096	7.003128486	0.2855016746	1.159401201
47	0.8203047484	0.4812180429	0.08180706729	6.031057008	5.914400278	-4.325518536	4.03361129	7.076023644	0.2806637865	1.132088618
48	0.837758041	0.4721385558	0.08026355449	6.168047699	5.981193581	-4.44489306	4.002199688	7.149620571	0.2759926032	1.10457751

To improve on this result, we combined our equations to cumulatively write out the complete expression of X_{BC} given the angle of the incline x.

a, b, c, d, e, and f are the parameters of the experiment – the coefficient of friction, the mass, etc.

$$-\frac{\sqrt{2}\cos(x)\sqrt{\frac{d(ab\sin(x)-abc\cos(x))}{a}}\left(\sqrt{2}\sin(x)\sqrt{\frac{d(ab\sin(x)-abc\cos(x))}{a}}-\sqrt{\frac{2d\sin^2(x)(ab\sin(x)-abc\cos(x))}{a}+2bf}\right)}{b}$$

The goal was to find the extreme values of this expression – this would occur whenever the derivative was 0. The derivative was calculated using an online derivative calculator. Next, the root was found using a graphing calculator. The optimal angle was thus when θ = 26.6500131 degrees. The distance when this occurred was 1.517843076 m. This proves that our spreadsheet was very close to the exact answer.

Extensions:

- One possible extension to this problem is to add air resistance, which would add variable acceleration based on position and mass to the projectile part of the equation when solving for distance. To solve this, an equation would be given in the problem that models the air resistance and calculus can be used to solve for velocity and distance where the puck lands.

- Another possible extension could allow the coefficient of friction on the ramp to vary. An expression for the coefficient of friction can be given, and the acceleration would change along the ramp – calculus must be used to determine the final answer, the distance the puck travels.

- Another extension could involve a ramp with variable incline instead of a straight drop. Then, the angle would vary with distance and the acceleration could be expressed as a function of distance (L). The goal would be to determine the optimal ramp shape to maximize the distance X_{BC}.