Question: How does the relationship between the elevation angle of an incline to the acceleration of a cart on it obey Newton's Second Law?

Hypothesis: The relationship between acceleration and g multiplied by sine of the elevation angle will be linear. The slope of the graph of acceleration vs. g multiplied by sine of the elevation angle, will equal the mass of the cart on the incline divided by the total mass of the system.

Strategy:

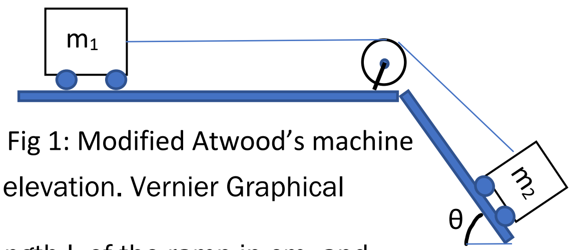

Fig 1: Modified Atwood's machine

- A rope connected cart 1 on a horizontal plane with cart 2 on an incline of θ degrees to form a modified Atwood's machine.
- The angle θ was varied by propping up the inclined plane on books to raise its starting height, and thus decrease its angle of elevation. Vernier Graphical Analysis was used to measure the resulting acceleration.
- θ was determined by measuring the height H of the setup and length L of the ramp in cm, and calculating $\theta = sin^{-1}(H/L)$. g sin(θ) was calculated from there using g = 9.8 m/s^2.
- The mass of both carts was not varied across experiments, with the cart 1 having a constant number weights inserted within – this lowered its acceleration to give a longer time to collect data.
- The acceleration of the cart 1 was graphed vs. g sin(θ) to ascertain that the slope was equal to the mass of cart 2 divided by the total mass of the system. The total mass includes the carts, added weights, and all other attachments – in other words, $m_1 + m_2$.

Data:

Mass of cart 2: **0.3021 kg**
Total mass of the system: **1.107 kg**

# Books	g sin(θ)	Acceleration (m/s^2)
0	5.90	1.530
1	4.90	1.344
2	4.26	1.171
3	3.61	0.970

The acceleration is an average of three trials

Analysis:

Figure 2 contains the free body diagrams with the forces in the setup.

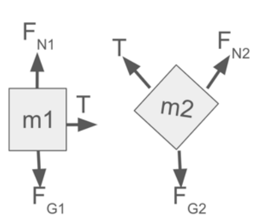

Figure 2: Free Body Diagrams

Friction and air resistance were not considered for this experiment, given that the cars are not significantly affected by these forces. On the second cart, tension acts parallel to the ramp and the normal force acts perpendicularly. Since the second cart stays on the ramp, forces perpendicular to it are balanced, so the only forces to consider on it are those that act parallel to it. Thus, only parallel gravitational force is included, which attenuates it by a factor of sin(θ). Treating rightward and downward forces as positive produces the following equations:

$$T = m_1 a$$

$$F_{G2} \sin\theta - T = m_2 a$$

Combining and substituting within these equations,

$$m_2 g \sin\theta = (m_1 + m_2)a$$

$$\frac{m_2}{m_1 + m_2} g \sin\theta = a$$

This asserts the theoretical linear relationship between acceleration and g sin(θ), where the slope is $\frac{m_2}{m_1+m_2}$, or the mass of the cart on the ramp divided by the total mass of the system. Figure 3 confirms this assertion:

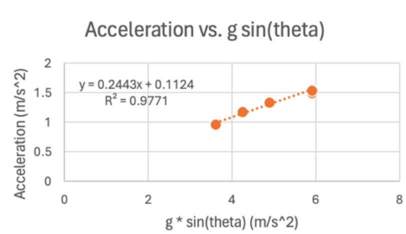

Figure 3: Experimental Data

The graph is certainly very linear with a high R^2 of 0.9771. The slope is 0.2443, which accords with the true value of $\frac{m_2}{m_1+m_2}$ based on measurement of 0.2730 – this means it is 10.6% smaller than expected. A smaller slope signals that acceleration was also lower than expected. A potential cause for this error could be that the rope connecting the two carts had a little slack – this would lag the first cart's reaction to forces acting on the second cart, and lower the measured acceleration. Friction on the carts' wheels could also have reduced its acceleration and thus the slope of the graph.