

WPI Physics Dept.
Intermediate Lab 2651
Thomson's experiment: Charge/Mass Ratio of the
Electron

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1 Background Material and Experimental Procedure

1.1 Introduction.

In this experiment, a beam of high-speed electrons is created by means of an "electron gun" inside a glass vacuum vessel, and its properties are investigated. The path of the beam is observed with the aid of a fluorescent screen.

Let $+z$ (toward the left) be the direction of the electron beam as it emerges from the electron gun. If an electric field \vec{E} is applied in the $+y$ direction (upward), transverse (perpendicular) to the axis of the beam, the electrons (bearing negative charge) are deflected from the axis in the $-y$ direction. If the electric field is uniform, then a constant transverse force is applied (hence the $-y$ acceleration) and the path of the electrons is a parabola.

Or, if a magnetic field \vec{B} is applied in the $+x$ direction (outward, toward the viewer) transverse to the axis, the electrons are deflected from the axis, initially in the $-y$ direction. If the magnetic field is uniform, then a transverse force of constant magnitude is applied which remains perpendicular to the electron velocity vector, and the path of the electrons is an arc of a circle.

This work will be in part a re-creation of conditions like those of the historic experiment by J.J. Thomson (1897, Cambridge University), who made use of electric and magnetic deflections of such an electron beam to determine the fundamental ratio e/m_e of electric charge to mass of the electron. Evaluation of this ratio was an important accomplishment, but his achievement was far more momentous than that: his result revealed unmistakably for the first time the existence of matter-particle nature of electrons, demonstrated by their motion in accordance with the laws of Newtonian mechanics. Thomson is credited with the "discovery" of the electron, the first particle to be identified as a constituent of atoms.

Reading: George Gamow, *Biography of Physics* (Harper & Row, 1961); Ch.VII, "The Law of Quantum," pp. 209-215.
(On reserve in Gordon Library)

1.2 Objectives.

The objective of the experiment is to measurement the ratio of charge/mass of an electron, in the spirit of the classic experiment of J.J. Thomson, who

first determined this ratio using crossed electric and magnetic fields. In performing the experiment you will learn the principles by which the electron beam is created and controlled, determine the electron kinetic energy and speed, observe and analyze the phenomena of electric and magnetic deflection, and learn how the strength of the transverse electric and magnetic deflecting fields can be determined.

NOTE: We shall see that with our simple version of the Thomson apparatus the accuracy of the e/m_e ratio we obtain is somewhat limited, but the *principles* can be clearly demonstrated.

1.3 Preparations.

Examine the apparatus; identify the “deflection tube” with its glass envelope, base with terminals for the electron-emitting filament electrode (this electrode is also called the *cathode*); electron-accelerating gun with external terminal for accelerating electrode (this electrode is also called the *anode*); electric deflection plates with the external terminals; and fluorescent observing screen with graticule (literally, “little grating” : measuring grid). Note the slit in the output end of the gun through which the accelerated electrons will escape into the vacuum space as a ribbon-shaped beam.

Identify the two *Helmholtz coils* for magnetic deflection, mounted outside on either side of the glass vacuum deflection tube. Note their terminals, labeled A and Z. Examine the current supply for the Helmholtz coils and the digital multimeter to be connected as an ammeter. Verify that its current-measuring terminals are in *series* with the Helmholtz coils, so that the same current passes through the meter and through each coil.

Examine the *accelerating-potential* supply: identify the line-voltage (on-off) switch; voltage and current dials; meters. Examine the *deflection-potential* supply: identify the line-voltage switch; 6.3 volt AC terminals for filament (electron emitter); high voltage + and – terminals and center tap; slide-wire variable high-voltage control; kilovolt (KV) meter.

With assistance by an instructor, check out the circuit, or if necessary reassemble the circuits for the filament and accelerating potential.

*****CAUTION*****

Whether you have connected the circuits yourself,
or have found them already connected, in either case ask an
instructor to help you verify the circuits before turning on
the power. The deflection tube is delicate and, if damaged,
cannot be repaired.

1.4 Observations.

To determine the speed v_0 of the accelerated electrons, we can use the *work-energy theorem*: relate the decrease of *potential* energy of the electron as it “falls” along the z -axis through the electric field inside the gun, to the increase in its *kinetic* energy.

$$\frac{1}{2}m_e v_0^2 = eV_{ac} \quad (1)$$

thus,

$$v_0 = \sqrt{\frac{2eV_{ac}}{m_e}} \quad (2)$$

Using the textbook value for e/m , calculate the speed v_0 . For perspective, compare it with some other electron speeds: the mean speed due to thermal kinetic energy $E_{kin} = (3/2)k_B T$ of an electron at room temperature $T = 294$ K (k_B = Boltzmann’s constant); the orbital speed of the electron in the lowest (ground) state, according to the “old” Bohr theory of the hydrogen atom ($E_{kin} = 13.6$ eV); also compare v_0 with the speed of light in empty space. Is v_0 so great that effects of *relativity* need to be considered, or is it not?

Note the polarity, + and –, of the connections to the deflection plates. For the first observation, the upper plate is to be +, the lower plate –; the center tap of the deflecting-potential supply is to be at the same potential as the *anode* of the electron gun.

*****CAUTION*****

Ask an instructor to verify the circuit and its center-tap, “common” and “ground” connections before turning on the apparatus.

With the slide-wire control, increase V_{defl} and observe the effect on the beam. Raise the voltage to a value such that the trajectory of the deflected beam passes close to the upper half of the of the graticule. Record the value of V_{defl} . Observe the shape of the trajectory and, without yet taking detailed point-by-point data at this time, observe and make note of its appearance.

Now reduce the deflection potential to its minimum; reverse the + and – connections of the deflection voltage supply. Again, raise V_{defl} so that the deflected beam occupies half of the graticule; observe the new value of V_{defl} and the new form of the trajectory.

Turn down the deflection-potential to its minimum setting, but do not turn off the deflection voltage supply.

Confirm the Helmholtz coils are connected to each other and to the current source through the digital ammeter. The terminals labeled Z on the two coils are to be connected together by a wire jumper; the terminals labeled A go the the rest of the circuit. Thus, the two coils are connected *in series* – that is, the same current passes through one coil, then through the other, and then through the meter. This assures that the two coils act equally to produce the magnetic field. Confirm that the current will flow in the same *sense* (clockwise or counterclockwise) in the two coils. (What would be the consequence if they didn't?)

Turn on the current source and increase the current I_{defl} ; observe the effect on the electron beam. [NOTE: Current in the range from zero to about 0.2 A should be ample for the work in this experiment.]

*****CAUTION*****

The current in the Helmholtz coil circuit should not exceed 2 A

Reduce the current I_{defl} to zero and unplug the connections to the current source, interchanging them to reverse the polarity of the connection to the Helmholtz coils. Turn on the current source again, increase I_{defl} , observe the effect, and compare with that observed above with the former polarity. Make note of this. From you observations so far, you should be able to tell whether the current goes clockwise or counterclockwise around the coils when it enters at terminal A. Record your conclusions in your lab notebook.

Reduce the current to zero when ready to go to the next section.

1.5 Measurements.

Electron deflection. Returning to the arrangement for electric deflection with polarity set to deflect the beam upward, set V_{defl} so that the deflected beam intersects the point on the graticule with the coordinates $z = 10$ and $y = 2$. An estimate of the deflection voltage can be made using the meter on the power supply. However, it is more accurate to use the 1000:1 voltage-reducing probe. The instructor will assist you in the operation of this probe. After measuring V_{defl} and recording the values of V_{ac} and V_{defl} , begin making point-by-point measurements of the trajectory. Make a table listing y and z coordinates of about 10 to 20 points on the beam trajectory, using the graticule units on the screen (the numbering units are centimeters) and including the entire length of the graticule in the z -direction. It may be helpful to turn out the room lights (when feasible) so as to improve the reading precision. Use a reading lamp as a work light.

Partners, take turns reading and recording. Notice the uncertainty due

to imperfect focus of the beam, and adopt a system for estimating the location of the “middle” of the path of the beam. Make notes of your estimates of the precision and the conditions contributing to uncertainty in the measurements. Interpolate by eye to obtain an extra digit between the finest graticule units.

You can obtain another estimate for the experimental uncertainty in the following way. Adjust the deflection voltage so that the trajectory passes through the point (10,2), as well as you can determine. Measure the deflection voltage using the probe, and record this value. Now, looking only at the screen, raise and lower the deflection voltage a bit and try to bring the trace back so it again passes through the point (10,2). After adjusting it as well as you can, measure and record again the value of the deflection voltage. By repeating this procedure several times, you will be able to do a statistical analysis of the uncertainty. In your report, compare and contrast these two methods for estimating the uncertainty.

Reduce V_{defl} to zero, reverse the polarity, reset V_{defl} , and repeat the procedure of measuring the trajectory of the deflected beam, as above.

Reduce V_{defl} to zero, change the accelerating potential V_{ac} from 2.0 kV to 2.5 kV, and under this new condition repeat the procedure of Parts 5.1 and 5.2, above. as before, set V_{defl} to make the deflected beam intersect the coordinates (10,2), and note the new value of V_{defl} .

Reduce V_{defl} to its minimum setting. Reset V_{ac} to 2.0 kV.

Magnetic deflection. Turn on the current source, raising the current I_{defl} so that the deflected beam intersects the point with coordinates $z = 10$, $y = 2$, as before. In reading the digital ammeter, notice carefully the number of digits which are fixed and the number which vary, if any. The varying digits: are they drifting steadily, or fluctuating about a stable value? Again make the table and list y and z coordinates of about 10 to 20 points on the beam path, including the entire range of z . Partners, take turns.

Reduce I_{defl} to zero, reverse the polarity, reset I_{defl} and repeat the procedure.

Change V_{ac} from 2.0 kV to 2.5 kV, and repeat the procedure of Parts 4 and 5 of the experimental checklist, section 3.

Return I_{defl} to zero, turn off the current source, and reset V_{ac} to 2.0 kV.

The experimental technique adopted by J.J. Thomson in his determination of e/m_e included applying *both* electric and magnetic deflecting fields, and adjusting the polarities and strengths of the two fields so that their effects canceled each other. We shall follow his example. Re-establish the condition of Part 5.1, with electric deflection in the $+y$ direction. Now turn on the current source and adjust the magnetic field so that deflection is re-

duced to zero, as well as you can. You will note that, due to conditions not being ideal, even with your best effort the beam does not follow the z -axis precisely. Take close note of this; estimate over what range of variation the beam deviates. Record your observations, and consider what conditions might be contributing to these deviations.

Nevertheless, make a “best” adjustment of I_{defl} to make the deflection zero as nearly as feasible; record V_{ac} , V_{defl} , and I_{defl} . Repeat several times; partners, take turns. Get a feeling for the uncertainty in this measurement by increasing the current from your “best” value until it results in a line that you would say is definitely *not* straight. Do this again, *decreasing* the current this time. These two values of current represent your extreme values, and halfway between them should be your “best” value. Repeat the procedure with the signs of the electric and magnetic deflecting forces reversed. Repeat the procedure with the accelerating potential V_{ac} set at 2.5 kV. Also, time permitting, repeat the procedure using a number of values for the electric deflection voltage. Since the current required to make the deflection zero is expected to be proportional to the deflection voltage (see section 9), as you can then make a graph of current vs deflection to get better statistics.

1.6 Helmholtz coils.

Hermann von Helmholtz (1821-1894), brilliant German scholar of many fields including physics, is remembered for many remarkable contributions, including the original statement of the principle of *conservation of energy*. In thinking about ways to improve electrical instrument, he devised a simple technique to produce a very uniform magnetic field – not very strong, and uniform only over a limited region but quite suitable for many instrument applications, including electron-beam deflection tube. The basic Helmholtz configuration is a pair of circular coils, each coil made of N turns of wire and having average radius R . The two coils are situated with their centers on the same axis (in our case, the x -axis), with their planes parallel. Steady electric current I passes through both coils in the same sense (according to the right-hand rule). If the distance D between the centers of the coils has the correct “official” Helmholtz value, then the magnetic field B_x on the x -axis at the point $x = x_0$ midway between their centers is very uniform. How uniform? Well, “flat,” in the sense that $\partial B_x / \partial x = 0$ and also $\partial^2 B_x / \partial x^2 = 0$ at the midpoint x_0 . This means that at x_0 the function $B_x(x)$ has zero slope and zero curvature, so it remains close to the maximum value $B_x(x_0)$. By virtue of the geometrical symmetry of the configuration and also by the spatial relations, dictated by Maxwell’s equations, among the components

of the magnetic field, B_x also remains close to its maximum value over the yz -plane as well as along x . It can be shown that the official Helmholtz value of D , the proper distance between the planes of the two coils to produce this condition of maximum uniformity of the field, is simply equal to the coil radius: $D = R$.

To determine B_x acting on our electron beam, we apply the *Biot-Savart law* and obtain B_x along the x -axis as a function of the current I . Suppose a *single* coil is located in the yz -plane with its center at the origin, $x = 0$. The field B_x at the origin is,

$$B_x(0) = \frac{\mu_0 N I}{2R}, \quad (3)$$

where, μ_0 is the *permeability of empty space*, $\mu_0 = 4\pi \times 10^{-7}$ MKSA units (henries per meter, or Hm^{-1}). Along the x -axis B_x decrease, compared to the above maximum, according to

$$B_x(x) = \frac{\mu_0 N I}{2} \frac{R^2}{(R^2 + x^2)^{3/2}}. \quad (4)$$

Now, according to Helmholtz's prescription, the second coil is placed parallel to the first with its center at $x = R$. The midpoint of the pair is at the point we call x_0 , namely $x = R/2$. (It helps to make a simple diagram of the arrangement, viewed with the coils on edge.) Then, with both coils contributing equally, we have

$$B_x(x_0) = \frac{\mu_0 N I}{R} \frac{1}{(5/4)^{3/2}}. \quad (5)$$

This relation holds quite accurately near the center of the Helmholtz coil configuration of our electron-beam apparatus. The number of the turns N is written on the Helmholtz coil; you should measure the radius is the coil yourself, making careful note of the uncertainty in this measurement.

1.7 Theory

Here we tersely review the theory as to how it applies to the experiment.

The Lorentz force is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (6)$$

and you have applied various combinations of E- and B-fields to electrons. The resultant trajectories are parabolic to a good approximation,

$$y = y_0 + v_{y0}t + \frac{1}{2}a_{y0}t^2 = y_0 + v_{y0} \left(\frac{z}{v_{z0}} \right) + \frac{1}{2}a_{y0} \left(\frac{z}{v_{z0}} \right)^2. \quad (7)$$

These can be fit to a second-order polynomial using MATLAB

$$y = y_0 + Bz + Cz^2, \text{ where} \quad (8)$$

$$C = \frac{a_{y0}}{2v_{z0}^2}. \quad (9)$$

For the three experiments the forces are

$$F = ma_{y0} = 2mv_{z0}^2 C = \begin{cases} qv_{z0}B_x & \text{B deflection} \\ q(E_y + v_z B_x) = 0 & \text{nulling} \\ qE_y & \text{E deflection} \end{cases}. \quad (10)$$

This is the master formula, upon which the calculations of q/m are based.

2 Pre-lab Exercises

Prepare before entering lab – day 1

- (a) Calculate the ratio e/m for the electron.
- (b) Determine the speed of electrons accelerated from rest through a potential difference of 2000 V. Repeat for 2500 V.
- (c) Calculate the magnitude of B required to cause 2000 V electrons to orbit on a circular path of radius 20 cm.

Prepare before entering lab – day 2

- (a) As requested in the instructions, determine the speed of thermal electrons whose kinetic energy is equal to $(3/2)kT$, $T = 300$ K, and k is the Boltzmann constant; repeat for electrons with kinetic energy equal to 13.6 eV.
- (b) In yesterday's set of exercises, you calculated the magnetic field strength needed to cause electrons to orbit in a circular path of radius 20 cm. Now calculate the current needed in a Helmholtz pair of coils, each of radius $R = 6$ cm and 320 turns per coil, to produce that magnetic field in the center region of the coil pair.
- (c) Had the result of the electric deflection part of the experiment with $V_{ac} = 2000$ V yielded a value of $C = 2 \text{ m}^{-1}$, what value of e/m would this lead to, using the B -value determined above? What fractional uncertainty in the final result is required to make this result consistent with the accepted value for e/m ?

3 Experimental Checklist

1. Trace through the circuitry of this experiment so that you can explain the purpose of every wire connection made to the e/m apparatus. Once you have explained this to the satisfaction of the instructor, you will be shown how to turn the equipment on and how to make the various adjustments and measurements.
2. Once the filament supply has been turned on and the accelerating voltage has been raised to 2000 V, the electron beam will be observed to trace out a path on the fluorescent screen. Apply a deflection current to the coils so that the beam traces out a path that passes through the point $(z, y) = (10, 2.4)$ cm. Spend some time practicing the measurement of beam coordinates on the grid. Note that the heavy lines form a cm grid, and that each cm is subdivided into 0.2 cm portions. With practice you should be able to develop a technique for estimating the coordinates of the beam center to 1/10 of a small division with an uncertainty of $\pm 1/10$ to $\pm 2/20$, depending on the beam size and fuzziness. That means that you will have y-readings of the sort 0.68 ± 0.02 cm, 2.16 ± 0.04 cm, etc. (See discussion of Uncertainties elsewhere; Why are there no odd numbers in the hundredths place?) Practice this until you and your partner are able to make measurements that are consistently within one uncertainty unit of each other. Consult with an instructor if have questions or problems.
3. With an accelerating voltage of 2000 V, measure the deflection current required to deflect the beam through $(10.00, 2.40)$ cm. Measure a total of about 9 coordinate points along the beam, using such z-values as: 10.00, 9.00, 8.00, 7.00, 6.00, 5.00, 4.00, 3.40, and 2.40 cm. **Repeat** for a downward deflection. **Repeat** both the upward and downward deflections for an accelerating voltage of 2500 V. **Be sure to measure the accelerating voltage and the deflection current both at the beginning and at the end of each set so as to see whether voltage or current drift is an issue.**

(You should try to get this far by the end of Day 1. With these measurements and a value for the coil radius - see Item 6 below - you now have enough to determine e/m by the magnetic deflection approach.)

4. With an accelerating voltage of 2500 V, measure the deflection voltage required to cause an electric field deflection of the beam through the

point (10.00, 2.40) cm. Again, measure about 9 coordinate points along the beam, using the same z-values as before. **Repeat** for a downward deflection with an accelerating voltage of 2500 V. **Repeat** both the upward and downward deflections for an accelerating voltage of 2000 V. **Be sure to measure the accelerating and deflection voltages, both at the beginning and at the end of each set in order to evaluate the amount of drift.**

5. With an accelerating voltage of 2500 V and an electric deflection through the point (10.00, 2.40) cm, measure the coil current required to null out the electric deflection. Because the null is not perfect and the beam never appears as a straight line, you should record what you believe to be reasonable upper and lower limits to the null current. **Repeat** for downward electric deflection and upward magnetic deflection. **Repeat** both null conditions for an accelerating voltage of 2000 V.
6. For your subsequent analysis, you will need values for several system parameters, such as spacing between the deflection plates, the radius of the coil pair, and the number of turns of wire in each coil. Use the centimeter grid inside the tube to get an estimate of the deflection plate spacing. Use a ruler and caliper to measure the inside and outside diameter of the coil, the coil width, and the average spacing (center-to-center) between the two coils. The number of turns in each coil is printed on the coil near the electrical connections.

4 Report Checklist

Based upon your measurements you will be able to determine the q/m ratio independently two different ways, which are for the magnetic deflection and the nulling experiment.

4.1 Magnetic deflection

From the first case of Eq. (10) it follows that

$$\frac{q}{m} = \frac{2v_{z0}C_{B.\text{defl}}}{B_x} = \frac{2C_{B.\text{defl}}}{B_x} \sqrt{\frac{2qV_{ac}}{m}}. \quad (11)$$

Square both sides and perform some algebra to get

$$\frac{q}{m} = \frac{8C_{B.\text{defl}}^2 V_{ac}}{B_x^2}. \quad (12)$$

Making use of Eq. (5) we finally arrive at a value for the ratio

$$\frac{q}{m} = \frac{125V_{ac}R^2C_{B.\text{defl}}^2}{8\mu_0^2N^2I_{B.\text{defl}}^2}, \quad (13)$$

where V_{ac} is the accelerating voltage, R is the average radius of the Helmholtz coils, N is the number of turns in each coil, $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$, I_{defl} is the current which deflects the beam and $C_{B.\text{defl}}$ is defined in Eq. (9).

4.2 Electric and null deflections

Here we start with a zero Lorentz force $E_y + v_z B_x = 0$. First, determine E_y from the last equality of Eq. (10), and make use of the accelerating Voltage, Eq. (1), to get

$$E_y = 4C_{E.\text{defl}}V_{ac}. \quad (14)$$

Then, use the formula for the Helmholtz coils Eq. (5) to obtain the ratio

$$\frac{q}{m} = \frac{125R^2E_y^2}{128\mu_0^2N^2I_{\text{null}}^2V_{ac}} \quad (15)$$

where V_{ac} is the accelerating voltage, R is the average radius of the Helmholtz coils, N is the number of turns in each coil, $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$ and I_{null} is the current which nullifies the effect produced by deflecting electric field.

4.3 Notes

Here are suggestions for how to structure your write-up:

1. Calculate e/m from each set of data (upward/downward deflection at various accelerating voltages) in each of the cases mentioned above. Find the average values of the e/m from the magnetic deflection data and the electric/null deflection data separately. Report these two values as your final results.

2. In the electric/null deflection part of your experiment, estimate the deflecting electric field $E_y = V_y/d$, where V_y is the deflecting voltage applied across the two plates and d is separation between the two plates. Calculate e/m using this value of the deflecting field and compare this value of e/m with the one obtained by using E_y from the data fitting.