Single Photon Interference

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(Dated: February 11, 2015)

Light is known to possess both particle-like and wave-like properties. In this paper, Richard Feynman's famous thought experiment on wave-particle duality was performed and compared with theoretical predictions. Light was allowed to pass through two slits and the intensity as a function of angle was measured. The results appeared to correspond well to the values expected if light behave like a wave even at intensities low enough that there is only a single photon in the apparatus at a given time. These results were then used to support the quantum theory of light.

INTRODUCTION

When scientists began making theories regarding the nature of light, they had to consider whether light is a particle or a wave. There is large amount of evidence that supports both the idea of light behaving as a particle and light behaving as a wave. This seemingly paradoxical notion came to be known as the wave-particle duality, a concept stating that light behaves both as a particle and as a wave. Serving as a vital idea for quantum mechanics, this concept doesnt explain why light behaves that way, but confirms that light travels as a photon but behaves as a wave depending on the event occurring. In order to observe this, we have performed Youngs double-slit experiment with the appropriate equipment to measure light intensity of interference patterns and observe single photons traveling through space.

Light, when made to pass through two slits forms a diffraction pattern in a way explained only by wave mechanics. On the other hand it turns out that if one uses this theory to predict the intensity of light as a function of wavelength for hot objects it varies greatly from what is observed. In order to reconcile theory with reality Max Plank hypothesized that the energy of light was not related to its intensity as one would expect from waves but is related to its wavelength. While this may not sound significant further investigation finds that energy may only be added to light in packets called quanta.

Development of the Duality of Light The debate whether light is a wave or particle started during the Greek time period. Aristotle suggested that light behaved like a disturbance in the element of air, similar to a wave traveling in water. Democritus, on the other hand, explained that everything that existed in the universe is made of indivisible atoms, particles that cannot be further broken down. The idea of light as a particle was widely accepted when Isaac Newton became well-know in the science community. Using his prism experiment, he explained that reflection of light is made possible only by corpuscles, particles that travel in straight lines in space. Although the hypothesis could explain reflection of light, it did not fare well with other phenomena such as refraction and interference. Around that time, the theory of light as a wave was supported by Christiaan Huygens Treatise on Light. It explains on how light vibrates up and down like a wave. However, due to Newtons prestigious reputation, the notion of light as particles was more commonly accepted in the scientific community. This dominance lasted until new discoveries provided accurate support for the wave theory. Youngs double slit experiment demonstrated a pattern of light interference that spreads out as a wave. Maxwells equations provide a detailed description of light as an electromagnetic wave. These two discoveries made the light wave theory more dominant than the light particle theory. However, it wasnt the end of the theory as Einstein published a report that explained the photoelectric effect. He theorized that light travel not as waves but as quanta or photons, packets of energy that carried a specific frequency. There is significant evidence for light both as a wave and a particle, and the duality of light aims to join the two theories.

PROCEDURE

The apparatus used consisted of a slender light tight metal box equipped with two linear slides attached to micrometers, one positioned midway down the box and the other near the end. A slit, called the detector slit, was placed on the the slide nearest the end and another much wider slit, called the blocker slit, was placed on the slide in the middle. A double slit with the widths of the slits measuring $85\mu m$ and the distance between their centers measuring $353\mu m$ was placed approximately 0.5 cm before the the slit in the middle. A 670nm wavelength laser diode and incandescent light source was placed in the box at the end opposite the detector slit and a photodiode and photomultiplier tube (PMT) was placed after the detector slit in such a way that the only light that could reach them came through the opening of the detector slit. Finally, a single slit, called the source slit, with a

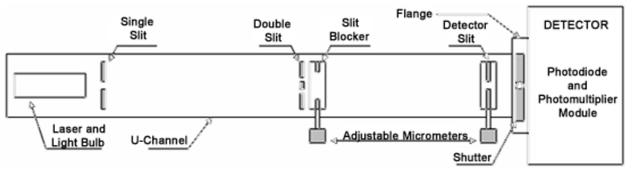


FIG. 1: Diagram of Apparatus
Obtained from:
http://teachspin.com/instruments/two_slit
/index.shtml

width measuring $85 \mu m$ was placed about 5cm after the incandescent light. A diagram of the full apparatus is displayed in figure 1.

Laser Diffraction Experiment

The first experiment was to measure the diffraction pattern of laser light through the setup using the photodiode detector. The system was first aligned by turning on the laser and centering its output on the source slit. The resulting single slit diffraction pattern was then used to align the double slits. They were positioned so that the central maximum covered both slits. The blocker and detector slits were then aligned vertically to the output of the double slit. Next the the blocker slit was moved using the micrometer so that it blocked both slits of the double slit. The position this happened at was then recorded. The positions at which it allowed light from the rightmost slit, both slits and the left most slits was recorded as well. These measurements are shown in Table I.

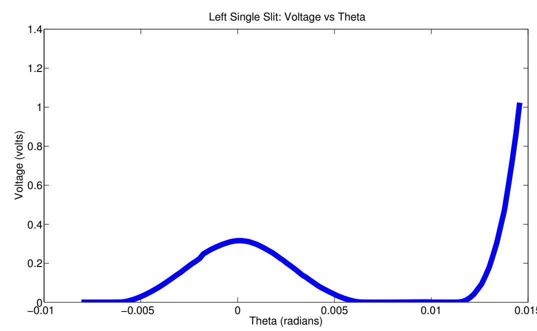
The laser was then shut off and the light tight box sealed. The voltage across the photodiode was measured as 8.0mV. The laser was then turned on again and for each position of the blocker slit previously found that allowed light through, the voltage across the photodiode was measured for a series of uniformly spaced positions of the detector slit. The photodiode voltage was then plotted against the position of the detector slit for each position of the blocker slit and these plots displayed in figures 2a, 2c and 2b.

TABLE I: Positions of Blocker Slit

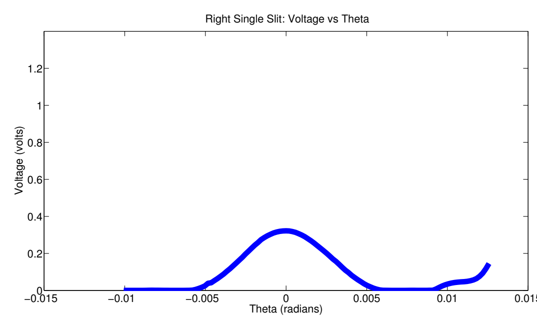
Blocker Position	Micrometer Pos (mm)
Both Slits Blocked Right	6.00
Right Slit Showing	5.50
Both Slits Showing	5.00
Left Slit Showing	4.65
Both Slits Blocked Left	4.00

Single Photon Diffraction Experiment

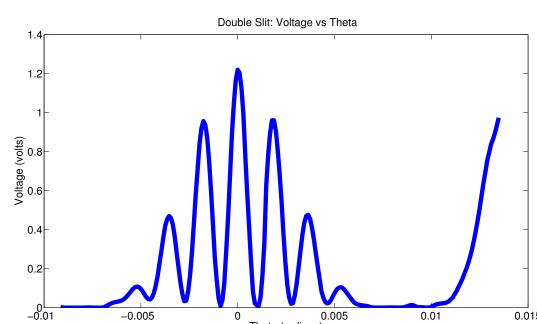
The next experiment was measuring the same diffraction patterns as before but using the incandescent light source and PMT at intensities such that only one photon is in the apparatus at any given time. The same alignment procedure used in the first experiment was followed but this time using light from the incandescent bulb. Next, an aperture was closed in front of the PMT blocking all light from the rest of the apparatus. The number of incident photons over 1s intervals was then measured from the PMT for 100 trials. Light from the bulb was then allowed to pass through the double slits and the detector slit was set to the position that resulted in the central maximum seen in figure 2c. The number of incident photons over 1s intervals was again collected. The results were then



(a) Left Slit



(b) Right Slit



(c) Double Slit

FIG. 2: Laser Diffraction Data: Voltage vs Theta

plotted as a histogram which is displayed in figures 9 and 10.

Next, a series of counts taken over 10 seconds was made and the aperture opened. With the incandescent light turned on a series of counts was taken over 10s for uniformly spaced positions of the detector slit with the blocker slit allowing light from both slits. The resulting counts were then plotted against the position of the detector slit and the resulting plots displayed in figure 3.

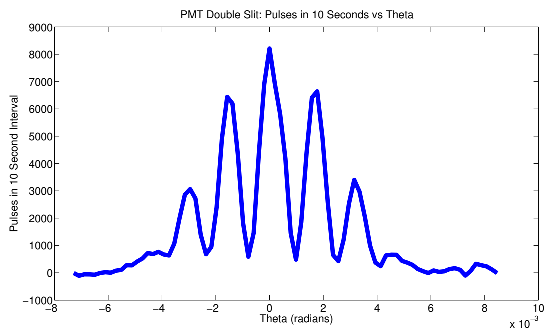


FIG. 3: Double Slit Incident Counts

METHODOLOGY AND THEORY

Measuring Intensities in Terms of Voltages

Throughout this experiment, the term intensity was used loosely. The values measured through the photodiode were not the exact intensities but voltages that are directly proportional to intensities. For a given intensity incident upon the sensitive area of the photodiode a current proportional to the it will be found through the device. By connecting this device to a resistor, who's voltage drop is directly proportional to the current flowing through it, the current may be inferred by measuring the voltage across the resistor. This voltage will then be directly proportional to the intensity of light and can be used as a stand in for the intensity in the experiments conducted. The manufacturer provided value for the diode's conversion rate was $0.4\frac{A}{W}$. Note that the denominator does not have the units of intensity. This is because the current is effectively proportional to the integral of incident intensities over the surface of the detector. For our setup however, the intensity as a function of position over the detector surface only changed by a scaling factor and so this integral comes out as another multiplicative factor. The resistor used had a nominal value of $22 \times 10^6 \Omega$.

Single Slit Diffraction

The theory that accompanies single slit diffraction suggests an intensity pattern with one large central maximum. Outside of the large central maximum, there are much smaller peaks that are significantly less detectable (and were not seen in this experiment). The equation that models the intensity of the diffraction pattern is

$$I = I_0 \frac{\sin^2(\frac{\delta}{2})}{(\frac{\delta}{2})^2} \tag{1}$$

with

$$\delta = \frac{2\pi a \sin(\theta)}{\lambda} \tag{2}$$

where I_0 is the measured central maximum intensity, λ is the wavelength of the light, and a is the width of the slit. The values specified in the lab manual were $\lambda = 670nm$ and $a = 85\mu m$, as previously mentioned in the procedure.

Double Slit Interference and Diffraction

The theory that governs the double slit diffraction is similar to that of the single slit diffraction with one extra complication. We will show first the mathematical description and then explain how it affects the visual result. The equation that models the intensity pattern of double slit interference and diffraction is

$$I = I_0 \frac{\sin^2(\alpha)}{\alpha} \cos^2(\beta) \tag{3}$$

with

$$\alpha = \frac{\pi a \sin(\theta)}{\lambda} \text{ and } \beta = \frac{\pi d \sin(\theta)}{\lambda} \tag{4}$$

where I_0 and a have the same meaning and d is the spacing between the centers of the double slits (specified to be $d = 0.353mm$). This equation is very similar to equation 1, but it has an extra cosine term. This extra cosine term causes an extra oscillation of the intensity peaks with higher frequency than the single slit. Visually, this causes the intensity of the double slit pattern to oscillate under the general envelope of the single slit. There are many more peaks and valleys for the double slit intensity pattern, and more than just the single central maximum was observable in the data. Figure 2 shows the general shape of the single and double slit intensity patterns. For now, ignore the right side of the graphs, as the data for higher values of θ is extraneous. This will be talked about later. Notice that figure 2c has higher frequency oscillations than figures 2a and 2b. This is the effect of the cosine term adding in an extra degree of oscillation of intensity.

Conversion of Distances to Angles

The measurements taken with regards to position along the detector were taken in millimeters, but the equations and the theory call for an angular variable in radians. Since the millimeter position from the micrometer was zeroed at the left side of the apparatus, we had to subtract the coordinate of the central maximum (so that it corresponded to the zero point). After subtracting this value, we used the equation

$$\theta = \frac{x}{L} \tag{5}$$

to convert the linear coordinate to an angular coordinate (which is derived using a small angle approximation such that $\sin(\theta) \approx \tan(\theta) \approx \theta$). In this equation, L represents the horizontal length from the double slit to the detector, which we measured to be 50.8cm. Table II shows conversion of theoretical raw data with intensity maximum at 5mm to angular coordinate θ.

TABLE II: Example Linear Data Converted to Angular Data

Raw Measurement x (mm)	Angle θ (rad)
0	-0.00984
1	-0.00787
2	-0.00591
3	-0.00394
4	-0.00197
5	0.00
6	0.00197
7	0.00394
8	0.00591
9	0.00787
10	0.00984

Single Photon Detection with Photomultiplier Tube

The theory associated with the detection of individual photon arrivals with the photomultiplier tube (PMT) takes more of a statistical nature than the theory of the laser diffraction and photodiode detection. For a given intensity of the light bulb and position of the detector, the number of photon events detected in a given interval will follow a Poisson probability distribution. Let N represent the set of readings taken in one second intervals. The most notable characteristic of the Poisson distribution is that the variance (which is the square of the standard deviation σ) is equal to the mean of N. Mathematically, the equation of a

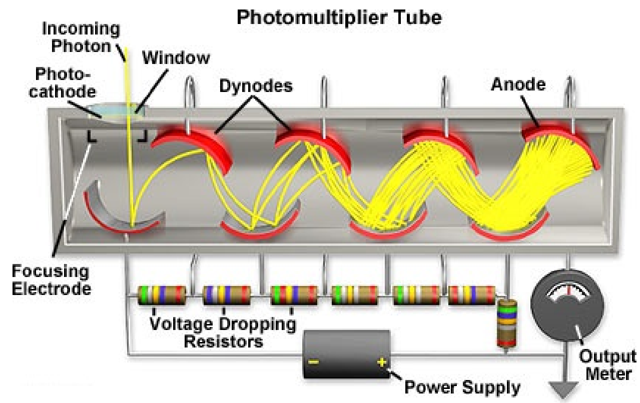


FIG. 4: Cutaway view of Photomultiplier Tube Obtained from: http://micro.magnet.fsu.edu/primer/digitalimaging /concepts/photomultipliers.html

Poisson distribution is

$$P(x, \lambda) = \frac{e^{-\lambda}(\lambda)^x}{x!} \text{ and } \lambda_N = \sigma_N^2 \tag{6}$$

where λ is the average of the set N and x is a discrete random variable. In this case, x is any nonnegative whole number corresponding to the number of photon events detected in the given time interval. For a given mean of λ photons detected, $P(x)$ will give the probability (out of 1) that there will be x photons detected. The second half of equation 6 addresses the previously mentioned fact that the variance of N is equal to the mean of N.

Operation of the Photomultiplier Tube (PMT) The operation of the PMT is much different than the operation of the photodiode. When one photon reaches the photocathode, an electron is released through the photoelectric effect. This electron then travels through the potential difference that is set in the PMT, causing the electron to be amplified and multiplied by several orders of magnitude. Once what is now a burst of electrons reaches the amplifier, it is converted into a positive voltage pulse which is seen on the oscilloscope and registered on the counter. The PMT does not have a perfect efficiency, however. There is about a 4% efficiency, meaning 4 times out of 100 a photon arriving at the photocathode will cause the release of an electron and register a pulse on the counter.

Due to the high voltage and the sensitivity of the photocathode, even modest levels of light risk damage to the apparatus, so great care was used when exposing the PMT to the light from the light bulb. An ordinary light bulb emits light from the entire spectrum of visible light, but we placed a green filter on the end of the bulb to severely limit the wavelengths that reach the PMT to a range of about $541nm$ - $551nm$. This limiting of wavelengths protects the PMT and brings the photon arrivals down to a manageable number to count. The general process of a PMT's operation is shown in figure 4.

DATA

Reflection of Light

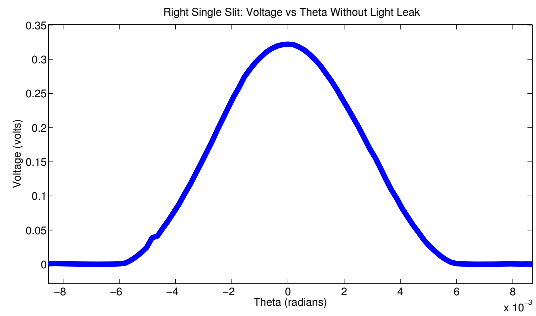
As we were doing the experiment, the data for the higher values of theta appeared flawed. Against theory, the right end of the box gave incredibly high voltage values, as seen in figures 2a - 2c, the right side of the graph tails up dramatically. Upon inspection of the apparatus, we observed that light from the laser was being reflected to that corner of the apparatus, causing uncharacteristically high readings when the detector was on that side. Due to this leak, the data on the extreme right sides of the graphs (where the θ values become most positive) is not realistic and will not be included in curve fitting approximations and data analysis of this section.

Single Slit Laser

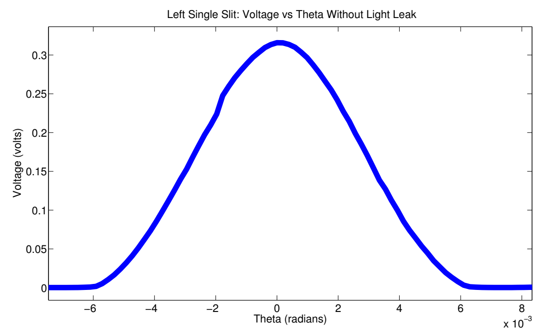
As previously mentioned, the theory that accompanies single slit diffraction suggests an intensity pattern with one large central maximum. In this portion of the experiment, $\lambda = 670 \pm 5nm$ and $a = 85\mu m$. As can be seen in figures 5a and 5b, there is a large central intensity maximum which fades off as θ strays away from zero. This matches the shape predicted by equation 1. Both individual slits of the double slit produce even, symmetrical curves which show that the two slits of the double slit are uniform.

Error Analysis for Single Slit Laser As previously stated, the specifications of the slit width and wavelength were $a = 85\mu m$ and $\lambda = 670nm$ respectively. Table III shows various curve fit parameters from the two MATLAB curve fits (one for each single slit, shown in figures 6a and 6b). The two columns in the table prefaced with "calculated" show the calculated value of the respective parameter with all other parameters assumed to be exactly to specification. The calculated λ values for the right and left vary from the expected $670nm$ by 15% and 13% respectively. Meanwhile, the variation of the a value for the right and left single slits are 18% and 15% respectively. The "RMSE" column shows the root mean square error of the curve approximation, and the order of magnitude of the RMSE suggests a very good curve fit. This low RMSE shows that the curve fit values fit the entire set of data without significant outlier subsets of data.

The uncertainty given for λ of the laser is $\pm 5nm$, so the calculated value of λ (while holding a to its given value) falls outside the recognized uncertainty. This can be explained by the effective variation of other parameters and uncertainty in the curve fit. First, if the source slit, double slit, and detector slit are not per-

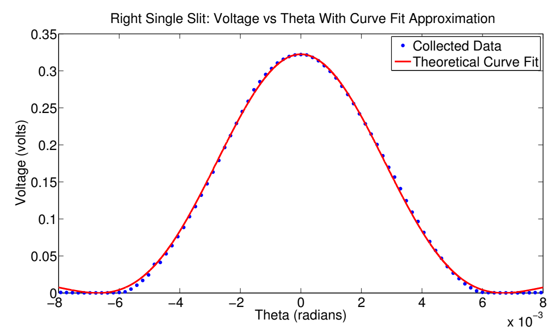


(a) Right Single Slit Pattern

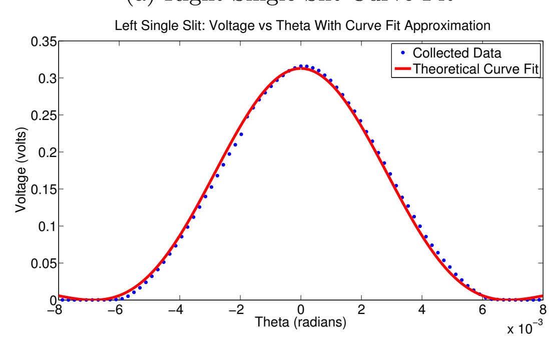


(b) Left Single Slit Pattern

FIG. 5: Single Slit Intensity Patterns Without Extraneous Light Reflection



(a) Right Single Slit Curve Fit



(b) Left Single Slit Curve Fit

FIG. 6: Single Slit With MATLAB Curve Fits

TABLE III: Curve Fit Data for Single Slit Intensity Pattern

Slit Side	Right	Left
I_0	0.3224	0.3128
$\frac{\pi a}{\lambda}$	468.8	457.9
RMSE	0.005765	0.005696
Calculated $\lambda(nm)$	569.6	583.2
Calculated $a(\mu m)$	100.0	97.7

fectly vertical, they will give the impression of being larger than their actual width. Therefore, the tilting of the three slits compounds to give an effective slit width greater than the specified value of $85\mu m$. Coupled with the uncertainty in the curve fit, it is realistic that the slit width could vary by up to 15%, as observed. The single slit intensity theory then fits the data quite well. There is also a chance that due to its width, the detector slit let in more light than is expected from the theory. Data was taken at specific points, which would require the detector slit to be infinitesimally thin (which it obviously is not). This would also cause the data to stray from theory and contributes to the variance observed between theory and fit of the data.

Double Slit Laser

For the double slit intensity pattern, due to the extra cosine term in equation 3, we could detect many more local maxima. Figure 7 shows the measured double slit intensity plotted versus θ.

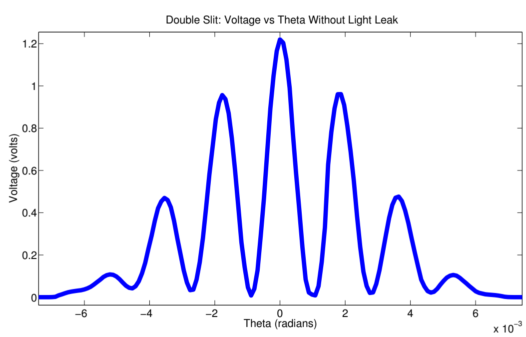


FIG. 7: Double Slit Intensity Pattern Without Extraneous Light Reflection

There is a defined outer envelope which matches the envelope of intensity for single slit diffraction. Also, the additional oscillation due to the cosine term is visible. The curve of the intensity is remarkably symmetric, which follows the theoretical prediction of equation 3.

Error Analysis for Double Slit Laser As seen in table IV, the MATLAB fit for the double slit interference intensity pattern (figure 8) has a RMSE that is higher than the RMSE of the single slit experiment. This higher RMSE is due to the more complex shape of the intensity pattern and the higher values of voltage being recorded. However, the RMSE remains quite small, showing still a good match to the data without significant outliers. The columns of table IV prefaced with "Calc" are the values that are calculated while holding all other parameters equal to their given specifications. The only difference is that the column "Avg Calc $\lambda(\mu m)$) is the average of the two λ values, one calculated from a in equation 4 and the other calculated from b.

TABLE IV: Curve Fit Data for Double Slit Intensity Pattern

I_0	1.196
$\frac{\pi a}{\lambda}$	434.8
$\frac{\pi d}{\lambda}$	1704
RMSE	0.03823
Avg Calc $\lambda(nm)$	632.5
Calc $a(\mu m)$	92.7
Calc $d(mm)$	0.363

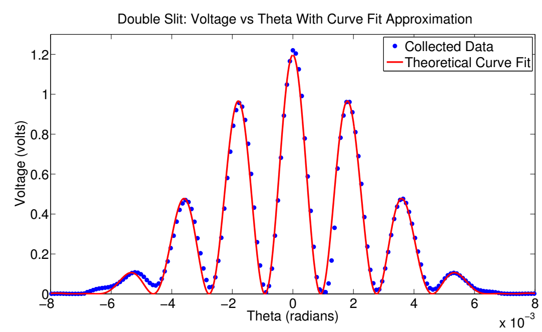
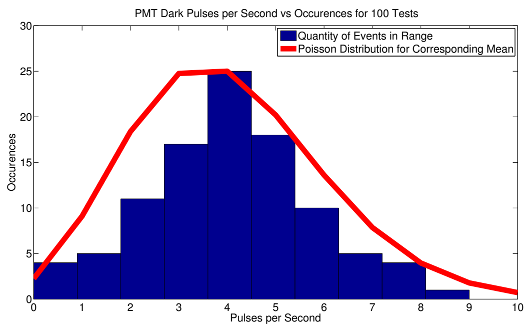
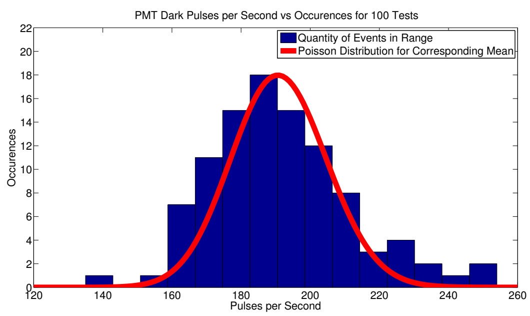


FIG. 8: MATLAB Fit for Double Slit Intensity

The table shows the calculated λ and a values to be slightly closer to the expected value than those from the single slit experiment. For this part of the experiment, λ varied from the specified value by 6%, while a varied by 9% and d varied by just 3%. It makes sense that the value of d varies so little because even if the double slit was tilted slightly, its effective spacing would not be greatly affected. Once again, it is likely that the tilting of the source slit, double slit, and detector slit caused an effective widening of the slit width a. This will still couple with the error in the curve fit to create the variance from the expected values. As with the single slit experiment, however, the error created by the effective widening of the slit and the error in the fit is of a realistic amount, and the data fits the theory as expected.

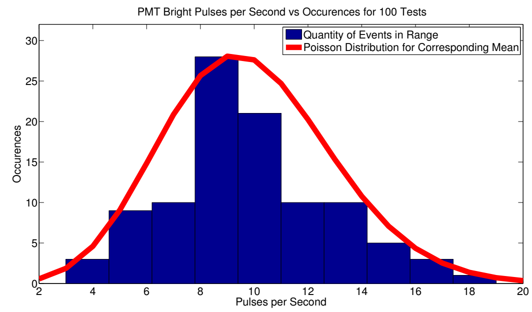


(a) Lower N_{avg}

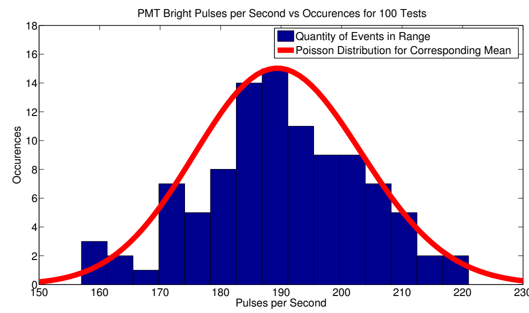


(b) Higher N_{avg}

FIG. 9: PMT Dark Incident Count Histograms



(a) Lower N_{avg}



(b) Higher N_{avg}

FIG. 10: PMT Bright Incident Count Histograms

PMT Light Bulb

Because of the statistical nature of the photon detection with the PMT and the filtered light bulb, figures 9 and 10 are histograms of the number of pulses recorded per second versus the number of occurrences in 100 trials. Two sets of tests (a high pulse count and low pulse count) were run for each state of the light bulb, lit and dark. The lit state was tested to observe the arrival of photons from the light bulb, while the dark state was tested to get an idea of the 'noise' of the PMT due to black-body radiation. The histograms for the lit bulb are shown in figure 10, and the histograms for the dark bulb are shown in figure 10

On the histograms, the Poisson distributions for the corresponding mean of each set of data is plotted in red along with the respective data. Since the Poisson distribution is normalized (so the probability of all events adds up to 1), the distributions on the graph are multiplied by constants to bring them into the order of magnitude of the histograms. Additionally. although the Poisson distribution takes only whole numbers as its independent variables, the individual points are connected to give the visual of a continuous distribution.

The histograms should follow the plotted Poisson distributions, and for the most part, that is what was observed. Table V shows the light bulb state, standard deviation, variance, and average of the four sets of tests done with the PMT. It is worth noting that the table follows the order of the histograms. For both of the tests with the lit bulb, the data appears to very closely fit a Poisson distribution. The variance is within 3% of the mean for both lit tests, which matches very closely to the theory. Because there was a finite number of data points taken, the data will not perfectly follow the predictions. Statistics just give a general idea of "what should, and will usually happen." For the dark light bulb tests, the data strayed from fitting to a Poisson distribution. The test in which $N_{avg} = 4.04$, the variance is within 10% of the mean, but for the test with $N_{avg} = 191.12$, the variance is more than 200% of the mean. This is likely explained by two factors. First, since the dark PMT noise is the result of black-body radiation, it likely will not fit to a Poisson distribution as well as the lit light bulb tests would. Additionally, since there is a large range of pulses detected in figure 9b (from about 140 pulses to 250 pulses), 100 tests is necessarily a large enough sample size. There is quite a large standard deviation and variance which would likely be lowered if more tests were taken in the data set. It is also by chance that the standard deviation ended up so large, which is the nature of statistics. Overall, the four histograms fit relatively well under the Poisson distributions, confirming that the release and arrival of photons follows a generally predictable distribution.

The Single Photon Nature of the Experiment There is a reason that the experiment is called "Single Photon Interference" that goes beyond the detection

TABLE V: Count Data for PMT

Light Status	σ	σ^2	N_{avg}
Dark	1.91	3.65	4.04
Dark	20.89	436.39	191.12
Lit	3.16	9.99	9.84
Lit	13.57	184.14	189.87

of one photon at a time. Looking back at figure 3, by summing all of the data points, we can calculate the total number of photons that arrive across the entire cross section of the apparatus in a 10 second interval. This can be used to calculate the total number of photon arrivals per second. This sum is 132520 pulses in ten seconds. Since the detector slit is $85\mu m$ wide and there were 81 readings taken across the cross section, approximately $6.9mm$ were covered across the cross section. Since the cross section is actually about double that distance, we will double the total number of pulses to account for missed detection area. This gives a total of 265040 pulses received in 10 seconds, or 26504 photons arriving per second. The lab manual gives an efficiency rating of about 4% for the PMT, meaning that if 26504 photons are detected each second, there are actually 25 times as many received each second, or 662600 photons received each second.

Using the knowledge that 662600 photons are arriving at the PMT each second, it can now be shown that there is actually only one photon in the apparatus most of the time. We measured the total length of the apparatus to be $98.6cm$, or $0.986m$. With this length, photons traveling at the speed of light $c = 2.998\text{x}10^8 \frac{m}{s}$ each take $3.29ns$ to travel through the apparatus. Meanwhile, if there are 662600 photons being released per second, assuming equal intervals of release gives that each photon is released $1590.21ns$ apart. Therefore, there is a photon in the apparatus for $3.29ns$ out of every $1590.21ns$, or 0.207% of the time. That means that there is no photon in the apparatus the other 99.793% of the time. Statistically, it is possible (yet extremely unlikely) that two photons are released within a small enough time interval to be in the apparatus at the same time, but generally there will only be one (if any) photons in the device at a time. This raises an interesting question about interference occurring only one photon, which will be talked about in the conclusion of this experiment.

CONCLUSION

Throughout the experiment, various data was collected that supports light both as a wave and as a particle. This dual nature is the basis of the wave-particle duality theory which has developed relatively recently.

Light as a Wave The interference and diffraction patterns measured in this experiment show a heavy support for light as a wave. Waves are known to diffract and interfere with themselves as they pass through slits, creating intensity variations which for the laser tests were measured to match the theoretical equations. The light from the laser diffracted and interfered as expected for both the single slit and double slit cases, which continues to support the wave nature of light. The single photon case of the light bulb also supported the wave nature of light. Particularly, figure 3 showed that the number of photons arriving per 10 second interval through the double slit also followed the same general pattern that the intensity of the laser did for the double slit tests. The consistency of the correlation between the theory and the data strongly supports that light is a wave.

Light as a Particle Light was supported to act as a particle in the experiment as well. The PMT was able to detect individual photon events through the photoelectric effect. The photoelectric effect is one of the largest proponents of light as a particle. Light particles (photons) with adequate energy arrive and cause the release of electrons which were amplified in the PMT. The PMT made it possible to actually measure the individual arrival of the photons, showing that photons arrive at not continuously (as the wave theory would suggest), but each at its own individual time. The nature of the PMT and the data taken from it strongly support that light is a particle.

Wave-Particle Duality The theory of wave-particle duality is supported through this experiment. Separate observations supported both the wave and particle like characteristics of the light in the experiment, which brings up many important questions. One of the loudest questions in the duality theory of light is "How can a photon interfere with itself?" Since it was shown that there was highly statistically likely that at most one photon was in the apparatus at a time, the notion of double slit interference seems paradoxical. How can one photon pass through both slits at once? That is one question that is not easily answered by classical notions of light, and the very same question is a large part of the field of quantum mechanics. The wave-particle duality theory of light attempts to explain what has been experimentally observed both in this experiment and others, but it does contain some seemingly paradoxical elements that must be explained using concepts of physics still being pioneered to this day.