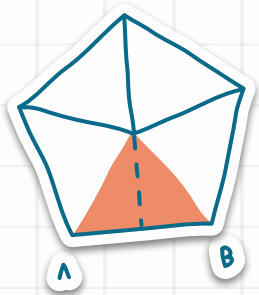


$$\bar{x} = \frac{\sum fx}{N}$$

Epsilon School

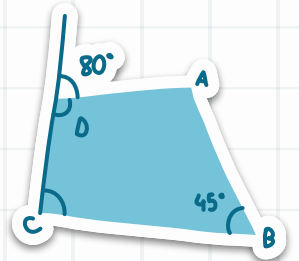


$$xy = ab^2$$



Claire Newcom, Dylan Whiting, Palak Yadav

$$g(x) = \sqrt{x(x-a)(x-b)}$$



Background

The Epsilon School of Mathematics and Science incoming sophomore class is larger than previous years, causing the student population to grow from 490 to 630. Because of this, seven new teachers will be hired.

$$A+B+C+D=360^\circ$$



Method

In this presentation we will show how we mathematically found out which departments should receive new teachers by using class data from previous years and attempting to keep the new and old student to teacher ratio similar.

$$A+B+C+D=360^\circ$$

Assumptions and Variables:

Assumptions:

- Students take 6 classes
- Class enrollment followed historical trends
- Consistently rounded down
- The dropout rate was consistent and no dropouts occur during senior year
- Teachers can teach students of any grade

Variables:

- Number of students in next year's 10th grade
- Number of students in next year's 11th grade
- Number of students in next year's 12th grade
- Number of students in each department next year
- Current class size

$$= \frac{\sum fx}{N}$$



$$ab^2$$

$$A+B+C+D=360^\circ$$

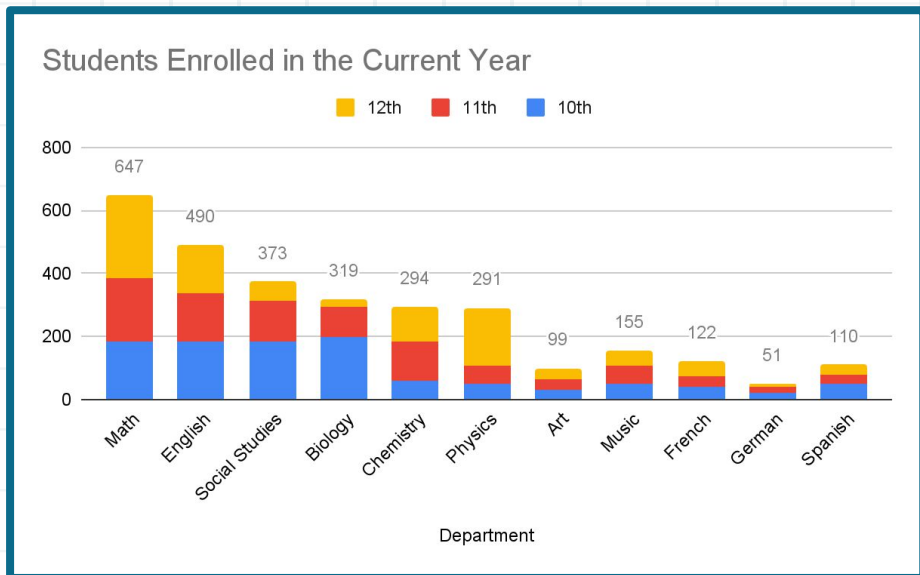
Conditions

| Criteria | Current Year | Next Year |
|----------------|--|--|
| Student Number | 490 | 630 Sophomores: 140 more than the graduating seniors |
| Faculty Number | 31 | 38 |
| Conditions | Size of the graduating class = graduating seniors + anyone who dropped out during the year | Incoming sophomore class is the size of the original size of the graduating class plus 140 |

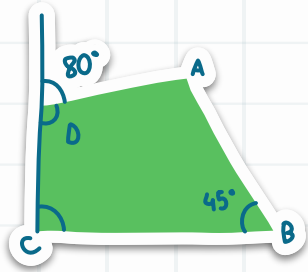
$$A+B+C+D=360^\circ$$

Current Numbers

| Department | Total | Faculty | Ratio(Student:faculty) |
|----------------|-------|---------|------------------------|
| Math | 647 | 6 | 108 |
| English | 490 | 5 | 98 |
| Social Studies | 373 | 5 | 75 |
| Biology | 319 | 4 | 80 |
| Chemistry | 294 | 3 | 98 |
| Physics | 291 | 3 | 97 |
| Art | 99 | 1 | 99 |
| Music | 155 | 1 | 155 |
| French | 122 | 1 | 122 |
| German | 51 | 1 | 51 |
| Spanish | 110 | 1 | 110 |



$$A + B + C + D = 360^\circ$$



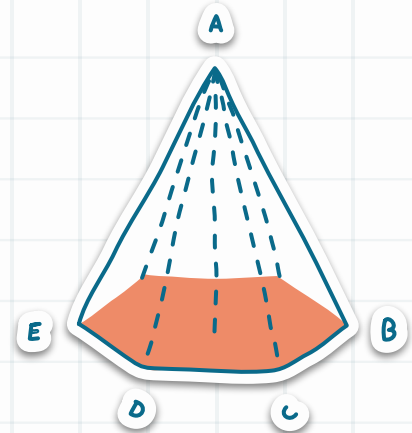
$$g(x) = \sqrt{x(x-a)(x-b)}$$

Problem

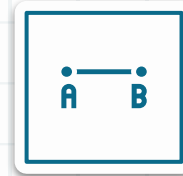
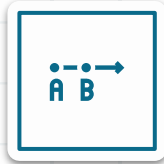
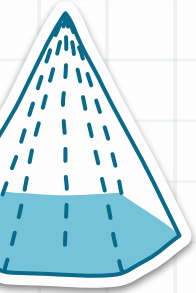
7 teachers are hired for next year to meet the increased enrolment.

How many teachers should each department receive?

$$\sqrt{\frac{a}{x}}$$



1. Finding Current Class Size



Process

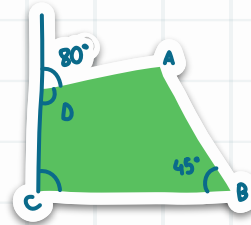
- Sum number of enrollments in each grade
- Assumed that each students takes 6 classes in a day
- Divide by 6 to identify the number of students in each grade

Reasoning

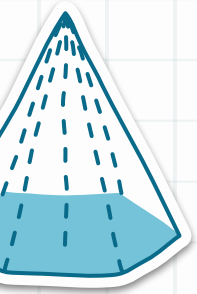
- Because each student takes six classes per day, if we add all of the classes taken up, and divide by six, that should give us the total number of students



$$g(x) = \sqrt{x(x-a)(x-b)}$$



1. Finding Current Class Size



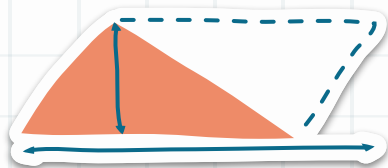
| Department | 10th | 11th | 12th |
|----------------|--------------|--------------|--------------|
| Math | 184 | 201 | 262 |
| English | 183 | 155 | 152 |
| Social Studies | 183 | 131 | 59 |
| Biology | 198 | 95 | 26 |
| Chemistry | 59 | 126 | 109 |
| Physics | 50 | 58 | 183 |
| Art | 31 | 33 | 35 |
| Music | 50 | 56 | 49 |
| French | 41 | 32 | 49 |
| German | 19 | 22 | 10 |
| Spanish | 51 | 26 | 33 |
| | | | |
| Sum | 1049 | 935 | 967 |
| Class Size | 174.8 | 155.8 | 161.2 |
| Rounded | 174 | 155 | 161 |



$$g(x) = \sqrt{x(x-a)(x-b)}$$

9

2. Using Drop Out Rates to Predict Class Size



Condition:

- 5% of the incoming class drop out prior to graduation

Size of the incoming class =
graduating senior class (+ any
students who dropped out during the
year

Current year:

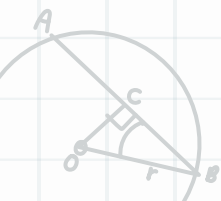
- Seniors: 161

Assumed no dropouts occur during senior year

- The senior year class originally started off with
169 students

(161 is 5% loss from 169)

- The incoming sophomore class is projected to
have **169 students + 140 additional = 309**
students



$$g(x) = \sqrt{x(x-a)(x-b)}$$



2. Using Drop Out Rates to Predict Class Size

Current Senior Year Class:

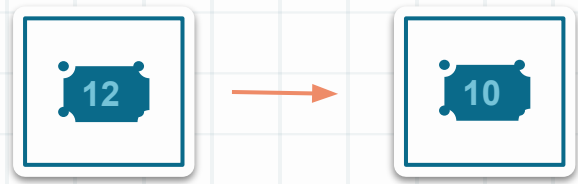
-Initial: 169 students

-Lose 8 students over 2 years

-Assumed even distribution of drop out → 4 students dropped out each year

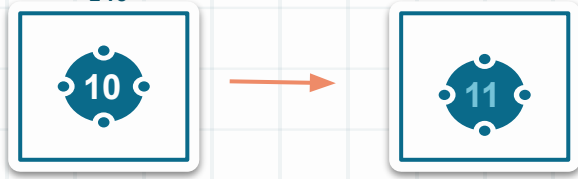
-2.367 % drop out rate each year

Grades Move Up → Lose 2.367% each year



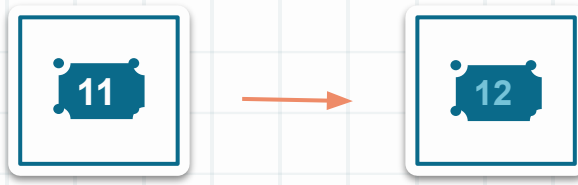
Original senior class size
+ 140

309



174

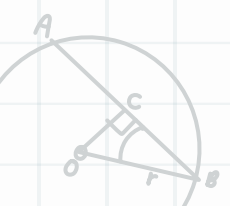
170



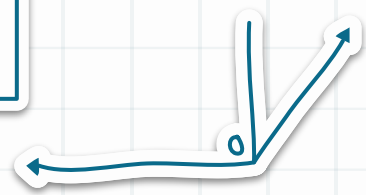
155

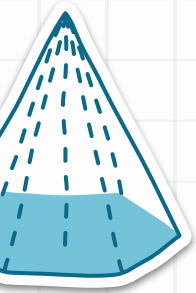
151

Total Students Next Year: 630



$$g(x) = \sqrt{x(x-a)(x-b)}$$





3. Finding a Ratio of Students to Predict Department Sizes



To find out the new number of students enrolled in classes in each of the departments, we used ratios.

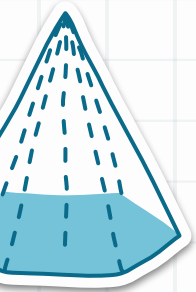
- Assumed that the same ratios of students would take classes in each of the departments over the years.
- For example, the ratio of students taking math in 10th grade in the current year is assumed to be equal to the ratio of students taking math in 10th grade next year.



Equation:

$$\frac{\text{students in department in current grade}}{\text{total number of current students in a grade}} = \frac{x \text{ students in department in new grade}}{\text{total number of new students in a grade}}$$

$$g(x) = \sqrt{x(x-a)(x-b)}$$



$$\frac{184 \text{ current 10th grade students enrolled in math}}{174 \text{ total 10th grade students}} = \frac{x \text{ 10th grade students enrolled in math next year}}{309 \text{ new 10th grade students}}$$

Example:

Solving for new
10th grade math:

$$x = \frac{(309 \text{ new students})(184 \text{ current 10th grade students enrolled in math})}{174 \text{ total 10th grade students}}$$

$$x = 326 \text{ 10th grade students enrolled in math next year}$$



$$g(x) = \sqrt{x(x-a)(x-b)}$$

Enrollment Numbers

| Department | 10th | New 10th | %Change | 11th | New 11th | %Change | 12th | New 12th | %Change |
|----------------|------|----------|---------|------|----------|---------|------|------------|---------|
| Math | 184 | 326 | 77.2 | 201 | 220 | 9.7 | 262 | 246 | -6.2 |
| English | 183 | 324 | 77.0 | 155 | 170 | 9.7 | 152 | 143 | -6.2 |
| Social Studies | 183 | 324 | 77.0 | 131 | 144 | 9.7 | 59 | 55 | -6.2 |
| Biology | 198 | 351 | 77.3 | 95 | 104 | 9.7 | 26 | 24 | -6.2 |
| Chemistry | 59 | 104 | 76.3 | 126 | 138 | 9.7 | 109 | 102 | -6.2 |
| Physics | 50 | 88 | 76.0 | 58 | 64 | 9.7 | 183 | 172 | -6.2 |
| Art | 31 | 55 | 77.4 | 33 | 36 | 9.7 | 35 | 33 | -6.2 |
| Music | 50 | 88 | 76.0 | 56 | 61 | 9.7 | 49 | 46 | -6.2 |
| French | 41 | 72 | 75.6 | 32 | 35 | 9.7 | 49 | 46 | -6.2 |
| German | 19 | 33 | 73.7 | 22 | 24 | 9.7 | 10 | 9 | -6.2 |
| Spanish | 51 | 90 | 76.5 | 26 | 29 | 9.7 | 33 | 31 | -6.2 |
| Class Size | 174 | 309 | | 155 | 170 | | 161 | 151 | |

$$y = (b \times a) - h$$

4. Student to Teacher Ratio

Current Year

- Using the data from the previous school year, we found the current student teacher ratio by dividing total number of students in a department by the amount of teachers for that department

$$\frac{\sum fx}{N}$$

Next Year

- Using the new department sizes found by assuming a constant ratio of subject size to class size, we found the new student teacher ratio by dividing the new total amount of students in the department by the amount of teachers before adding the new 7

| Department | Current S:T Ratio | New S:T Ratio |
|----------------|-------------------|---------------|
| Math | 108 | 132 |
| English | 98 | 127 |
| Social Studies | 75 | 105 |
| Biology | 80 | 120 |
| Chemistry | 98 | 115 |
| Physics | 97 | 108 |
| Art | 99 | 124 |
| Music | 155 | 195 |
| French | 122 | 153 |
| German | 51 | 67 |
| Spanish | 110 | 149 |

ABCD

$$\frac{a}{\sqrt{x}}$$

5. Distributing the Teachers

Finding the Difference

- We then found the percent difference between the two ratios for each subject using the percent increase formula

$$\frac{\sum fx}{N}$$

Distributing the Teachers

- Teachers were added to the departments in order of highest difference until all percent difference was $\leq 30\%$



= subject with a teacher added



| Department | % Increase w/o Added Teachers | % Increase w/ Added Teachers |
|----------------|-------------------------------|------------------------------|
| Math | 22 | 5 |
| English | 30 | 8 |
| Social Studies | 40 | 17 |
| Biology | 50 | 20 |
| Chemistry | 17 | 17 |
| Physics | 11 | 11 |
| Art | 25 | -37 |
| Music | 26 | -37 |
| French | 25 | -16 |
| German | 30 | 30 |
| Spanish | 36 | -9 |



$$c - \frac{b}{2}$$

$$\sum = 0$$



5. Distributing the Teachers

The departments that received teachers are as follows:

| Department | Teachers allocated |
|----------------|--------------------|
| Math | 1 teacher |
| English | 1 teacher |
| Social Studies | 1 teacher |
| Biology | 1 teacher |
| Art | 1 teacher |
| Music | 1 teacher |
| French/Spanish | 1 teacher |

$$\frac{\sum fx}{N}$$

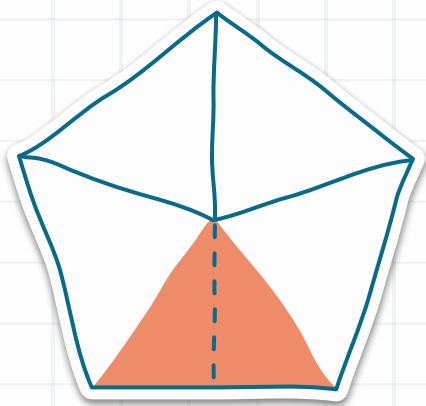
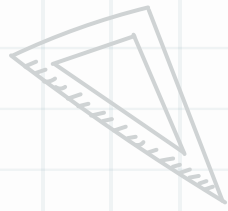


$$c - \frac{b}{2}$$



$$ax^2 + bx + c = 0$$

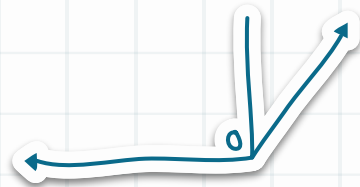
$$A+B+C+D=360^\circ$$



Analysis

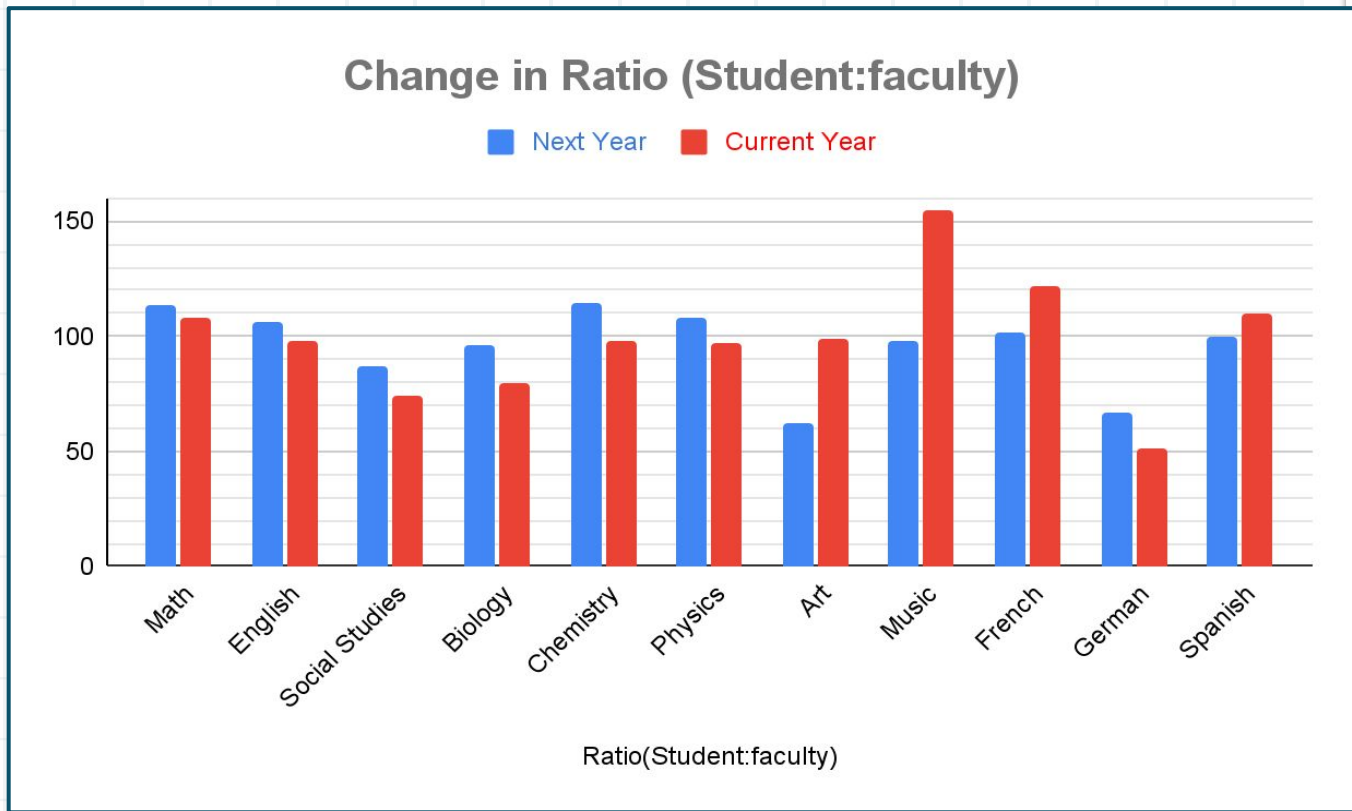


$$\left(\frac{4+4}{4^3}\right) \left(\frac{4(4^0+4+4^2)4^2}{4^3}\right) xy=ab^2$$





Student:Faculty ratio was maintained close to original or reduced and we consider that a positive



$$A+B+C+D=360^\circ$$

$$g(x) = \sqrt{x(x-a)(x-b)}$$



Checking Work

- We reinforced our answer by using a different method of finding class size
- By assuming each student took exactly one English class per year, we used department enrollment in english for class sizes
- % increase in classe sizes all stayed below 30% with our answer

$$A+B+C+D=360^\circ$$

$$g(x) = \sqrt{x(x-a)}(x-b)$$

| Department | % Increase w/ Added Teachers |
|----------------|------------------------------|
| Math | 2% |
| English | 4% |
| Social Studies | 11% |
| Biology | 13% |
| Chemistry | 14% |
| Physics | 9% |
| Art | -40% |
| Music | -39% |
| French | -19% |
| German | 26% |
| Spanish | -14% |





Validating Answer

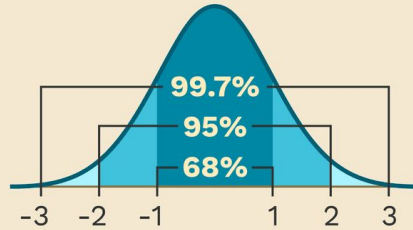
Calculating Standard Deviation

$$S_x = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

n = The number of data points

x_i = Each of the values of the data

\bar{x} = The mean of x_i



Normal Distribution Curve

ThoughtCo.

$$A + B + C + D = 360^\circ$$

$$g(x) = \sqrt{x(x-a)(x-b)}$$

| | Current Year | Next Year |
|--------------------|--------------|-----------|
| Math | 108 | 113 |
| English | 98 | 106 |
| Social Studies | 75 | 87 |
| Biology | 80 | 96 |
| Chemistry | 98 | 115 |
| Physics | 97 | 108 |
| Art | 99 | 62 |
| Music | 155 | 98 |
| French | 122 | 102 |
| German | 51 | 67 |
| Spanish | 110 | 100 |
| Mean | 99 | 96 |
| Standard Deviation | 27 | 17 |



Analysis

- The average student:teacher ratio remained about the same
- The standard deviation improved
- Infer that class sizes will remain relatively the same or better

| | Current Year | Next Year |
|--------------------|--------------|-----------|
| Math | 108 | 113 |
| English | 98 | 106 |
| Social Studies | 75 | 87 |
| Biology | 80 | 96 |
| Chemistry | 98 | 115 |
| Physics | 97 | 108 |
| Art | 99 | 62 |
| Music | 155 | 98 |
| French | 122 | 102 |
| German | 51 | 67 |
| Spanish | 110 | 100 |
| Mean | 99 | 96 |
| Standard Deviation | 27 | 17 |

$$A+B+C+D=360^\circ$$

$$g(x) = \sqrt{x(x-a)(x-b)}$$

$$E=mc^2$$

Strengths and Weaknesses

$$\frac{a}{\sqrt{x}}$$

Strengths

- Few assumptions needed to be made
- Checked that model worked with different class size assumptions

Weaknesses

- Assuming no dropouts can occur during senior year can be inaccurate
- Does not take into account how the school will accommodate students in future years
- Class enrollment may not always follow historical trends

$$\frac{\sum fx}{N}$$

$$ax^2 + bx + c = 0$$

$$x = -\sqrt{\frac{b^2}{c}} + c - \frac{b}{2}$$



Error Analysis

- Each grade has different interests and therefore may not have the same ratio of department to class size, which was not accounted for in our model
- In reality, students drop out throughout each year. In our model, we assume that no students drop out during senior year which would most likely not be the case
- Our class size assumed all students took exactly 6 classes every day which may not be the case as some may have study blocks
 - This error was corrected by trying out model with different assumptions made for class sizes

$$= \frac{\sum fx}{N}$$



$$ab^2$$

$$A+B+C+D=360^\circ$$



Thank you!

Extra gratitude to Mrs. Burns and the XYZ group!

