Study Guide 5

10.3 The radiative efficiency is $\eta = W_r/(W_r + W_{nr})$, and setting $1/\eta = 5$, we have

$$5 = \frac{W_r + W_{nr}}{W_r} = 1 + \frac{W_{nr}}{W_r}$$

so $W_r = W_{nr}/4 = 10^8/4 = 2.5 \times 10^7\ \mathrm{s}^{-1}$. Solving Eq.(10-10) for the hole concentration gives

$$p = \frac{W_r}{B_r} = \frac{2.5 \times 10^7\ \mathrm{s}^{-1}}{7.2 \times 10^{-10}\ \mathrm{cm}^3/\mathrm{s}} = \boxed{3.5 \times 10^{16}\ \mathrm{cm}^{-3}}$$

or $n = 5.6 \cdot 10^{20}\ \frac{\mathrm{cm}^4}{s}$

Note: Auger rate is $6 \cdot 10^{9}\ s^{-1}$ assuming $p \approx n$ so is negligible

10.4 (a) The efficiency will be maximized when $d\eta_i/dn = 0$, or equivalently, when $d/dn(1/\eta_i) = 0$. Using Eq.(10-13) we have

$$\frac{d}{dn}\left(\frac{A_{nr}}{n} + B_r + C_A n\right) = 0$$

$$-\frac{A_{nr}}{n^2} + C_A = 0$$

or $\boxed{n = \sqrt{A_{nr}/C_A}}$.

(b) Putting in the values from Example 10-2, we have

$$\text{GaAs: } n = \sqrt{\frac{10^7\ \mathrm{s}^{-1}}{5 \times 10^{-30}\ \mathrm{cm}^6/s}} = \boxed{1.41 \times 10^{18}\ \mathrm{cm}^{-3}}$$

$$\text{InGaAs: } n = \sqrt{\frac{10^7\ \mathrm{s}^{-1}}{1 \times 10^{-28}\ \mathrm{cm}^6/s}} = \boxed{3.16 \times 10^{17}\ \mathrm{cm}^{-3}}$$

(c) The efficiency is

$$\eta_i = \frac{B_r n}{A_{nr} + B_r n + C_A n^2}$$

which for GaAs becomes

$$\frac{(7.2 \times 10^{-10}\ \mathrm{cm}^3/\mathrm{s})(1.41 \times 10^{18}\ \mathrm{cm}^{-3})}{10^7\ \mathrm{s}^{-1} + (7.2 \times 10^{-10}\ \mathrm{cm}^3/\mathrm{s})(1.41 \times 10^{18}\ \mathrm{cm}^{-3}) + (5 \times 10^{-30}\ \mathrm{cm}^6/\mathrm{s})(1.41 \times 10^{18}\ \mathrm{cm}^{-3})^2}$$

or $\boxed{\eta_i = 0.98.}$ For InGaAs, the efficiency is

$$\frac{(4 \times 10^{-11}\ \mathrm{cm}^3/\mathrm{s})(3.16 \times 10^{17}\ \mathrm{cm}^{-3})}{10^7\ \mathrm{s}^{-1} + (4 \times 10^{-11}\ \mathrm{cm}^3/\mathrm{s})(3.16 \times 10^{17}\ \mathrm{cm}^{-3}) + (1 \times 10^{-28}\ \mathrm{cm}^6/\mathrm{s})(3.16 \times 10^{17}\ \mathrm{cm}^{-3})^2}$$

or $\boxed{\eta_i = 0.39.}$

10.6 (a)The junction width is due mostly to the depletion region on the lightly doped side, which in this case is the p side. The acceptor concentration in MKS units is $10^{20}\ \mathrm{m}^{-3}$. Eq.(10-20) then gives

$$d \simeq \sqrt{\frac{2\epsilon_r \epsilon_0 V_0}{e N_A}} = \sqrt{\frac{2(11.9)(8.85 \times 10^{-12})(0.56)}{(1.6 \times 10^{-19})(10^{20})}} = \boxed{2.71 \times 10^{-6}\ \mathrm{m}}$$

$\epsilon = 1.053 \cdot 10^{-10}\ \mathrm{F/m}$

where $\epsilon_r = 11.9$ is the relative dielectric constant for silicon.

(b) The maximum field occurs at $x = 0$, and has magnitude given by Eq.(10-17) as

$$E_{max} = \frac{eN_A d}{\epsilon} = \frac{(1.6 \times 10^{-19})(10^{20})(2.71 \times 10^{-6})}{(11.9)(8.85 \times 10^{-12})} = \boxed{4.13 \times 10^5 \text{ V/m}}$$

or, $V_0 = \frac{1}{2} E_{max} d$

$E_{max} \approx \frac{2V_0}{d}$

10.10 (a) From the given GaAs fluorescence wavelength, the bandgap energy of GaAs is

$$E_g \simeq \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{870 \times 10^{-9}} = 2.286 \times 10^{-19} \text{ J} = \underline{1.429 \text{ eV}}$$

In comparison, the photon energy emitted in the quantum well is

$$h\nu = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{830 \times 10^{-9}} = 2.396 \times 10^{-19} \text{ J} = \underline{1.498 \text{ eV}}$$

The energy difference is $h\nu - E_g = 1.1 \times 10^{-20}$ J. Using Eq.(10-24) with effective masses $m_e^* = 0.067m$ and $m_h^* = 0.48m$ (m is the free electron mass), we have

$$h\nu - E_g = \frac{h^2}{8d^2}\left(\frac{1}{m_e^*} + \frac{1}{m_h^*}\right) = \frac{17h^2}{8md^2}$$

Solving for the quantum well thickness gives

$$d^2 = \frac{17(6.63 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(1.1 \times 10^{-20})} = 9.33 \times 10^{-17} \text{ m}^2$$

or $d = 9.7 \times 10^{-9}$ m $= \boxed{9.7 \text{ nm.}}$

(b) For the quantum well levels to be bound, the band gap in the surrounding $Al_xGa_{1-x}As$ material must be greater than the photon energy $h\nu$. Using Eq.(10-9) we set

$$1.424 + 1.427x + 0.041x^2 > 1.498 \text{ eV}$$

and solving this gives $\boxed{x > 0.052.}$

11.3 (a) From Eq.(11-13), the response time is

$$\tau = \frac{1}{2\pi f_e} = \frac{1}{2\pi(80 \times 10^6)} = 1.99 \times 10^{-9} \text{ s}$$

The modulated amplitude is then determined from Eq.(11-12) to be

$$P_{mod} = \frac{10 \text{ mW}}{\sqrt{1 + (2\pi[250 \times 10^6][1.99 \times 10^{-9}])^2}} = \boxed{3.05 \text{ mW}}$$

(b) The carrier lifetime is $\boxed{\tau \simeq 2 \text{ ns.}}$

11.6 The geometry is similar to that of Fig. 11-5, except that the air side is replaced by glass of index $n_2 = 1.5$. Taking $n_1 = 3.6$ for GaAs, we first calculate the critical angle to be $\theta_c = \sin^{-1}(1.5/3.6) = 24.6°$. Light emitted within a cone of this half-angle is transmitted into the glass material. Using Eq.(A-2), this cone subtends a solid angle $\Omega = 2\pi(1 - \cos 24.6°) = 0.571$ sr. Assuming near normal incidence for the majority of the light passing through the boundary, the fraction of light reflected is given by Eq.(2-14),

$\theta_c = .429$ rad

$\pi \theta_c^2 = .579$ sr

$$R \approx \frac{(3.6 - 1.5)^2}{(3.6 + 1.5)^2} = 0.1695$$

and the corresponding fraction transmitted is $T = 1 - R \simeq 0.83$. The fraction of generated light that is emitted into the glass is therefore

$$\eta_{ext} = \frac{\Omega}{4\pi} T \simeq \frac{0.571}{4\pi}(0.83) = \boxed{3.77 \times 10^{-2}}$$

Comparing this with the results of Example 11-2, we see that more light is emitted into the second medium when the second medium has a higher index of refraction.

11.7 Consider the geometry of Fig. 4-1, in which a ray is incident from a medium of index n_0 and strikes the fiber end with angle α relative to the surface normal. We will take the incident medium to be GaAs, with $n_0 = 3.6$. Eq.(4-2) gives the maximum angle α_{max} for which light that enters the fiber core will be guided. Therefore we have $\alpha_{max} = \sin^{-1}(NA/n_0) = \sin^{-1}(0.25/3.6) = 3.98° = 0.0695$ rad. For such small angles, the solid angle is well approximated by Eq.(A-3), so $\Omega \simeq \pi\alpha^2 = \pi(0.0695)^2 = 0.01517$.

The Fresnel transmission through the boundary is the same as given in Problem 11.6, so $T \simeq 0.83$. The fraction of generated light that is emitted into guided modes of the fiber is therefore

$$\eta_{ext} = \frac{\Omega}{4\pi} T \simeq \frac{0.01517}{4\pi}(0.83) = \boxed{1.00 \times 10^{-3}}$$

This is a much smaller fraction than was found in Problem 11.6, because most of the light emitted into the glass of Problem 11.6 propagates at an angle where total internal reflection at the core-cladding boundary does not occur.

11.9 (a) From Eq.(11-16) we have

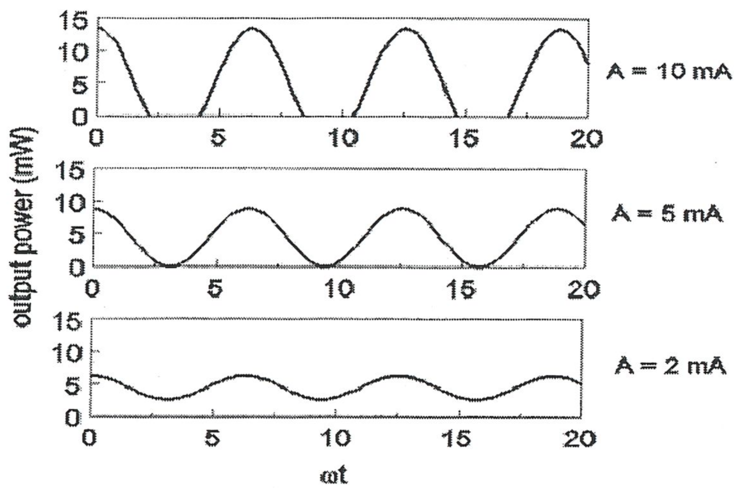
$$\beta_s = \frac{P_{out}}{i - i_{th}} = \frac{18 \times 10^{-3}}{(30 - 10) \times 10^{-3}} = \boxed{0.9 \text{ W/A}}$$

(b) The output power at $i = 40$ mA is $P_{out} = \beta_s(i - i_{th}) = (0.9)(40 - 10) \times 10^{-3} = 27 \times 10^{-3} \text{ W} = \boxed{27 \text{ mW.}}$

(c) The output power has time dependence

$$P_{out}(t) = \beta_s[i(t) - i_{th}] = (0.9)[15 + A\cos\omega t - 10] = (0.9)[5 + A\cos\omega t] \text{ mW}$$

with the additional condition that P is always positive (and therefore negative P in the above expression is interpreted as zero output power). With $A = 10$ mA, the output power is zero during a portion of the oscillation period, as shown in the top graph below. This is referred to as "clipping" of the waveform. For $A = 5$ mA, the power goes smoothly to a minimum of zero, remaining at or above threshold at all times. For $A = 2$ mW the power oscillates with amplitude 1.8 mW about an average of 4.5 mW.



11.10 (a) The change in pump power above threshold is $V_d\Delta i$, and so the slope efficiency is

$$\eta_s = \frac{\Delta P_{out}}{\Delta P_{in}} = \frac{\Delta P_{out}}{V_d\Delta i} = \frac{\beta_s}{V_d} = \frac{0.9 \text{ W/A}}{2.5 \text{ V}} = \boxed{0.36}$$

(b) For $i = 20$ mA, $P_{out} = (0.9)(20 - 10) \times 10^{-3} = 9 \times 10^{-3}$ W, and $P_{in} = V_d i = (2.5)(20 \times 10^{-3}) = 50 \times 10^{-3}$ W. The electrical-to-optical conversion efficiency is then $\eta = P_{out}/P_{in} = 9/50 = \boxed{0.18}$.

11.12 (a) The mode spacing is

$$\delta\nu = \frac{c}{2L} = \frac{3 \times 10^8}{2(0.15)} = 1 \times 10^9 \text{ Hz} = 1 \text{ GHz}$$

In a range of 1.7 GHz, two modes could potentially lase simultaneously.

(b) For GaAs, we take the index as $n = 3.6$, and calculate the wavelength spacing between modes to be

$$\delta\lambda = \frac{\lambda^2}{2nL} = \frac{(850 \times 10^{-9})^2}{2(3.6)(1 \times 10^{-3})} = 1.00 \times 10^{-10} \text{ m} = 0.10 \text{ nm}$$

The number of modes that could potentially lase within a 5 nm wavelength range is then $\approx 5/(0.1) = \boxed{50}$.