Problems Study Guide 4

7.2 Develop an approximate expression for the fraction of light lost due to lateral offset in a multimode fiber by expanding Eq. (7-1) in powers of δ/a. Assume $\delta/a \ll 1$ and keep just the lowest-order terms in δ/a.

7.3 Two multimode fibers with $a = 25\ \mu\text{m}$ are joined together, with a lateral offset $\delta = 1.5\ \mu\text{m}$. Use Eq. (7-1) to calculate the coupling efficiency and the fraction of light lost. Compare this with the result obtained using the approximate expression derived in Problem 7.2.

7.7 A single-mode fiber has a mode-field diameter of 8.5 μm. Determine the lateral offset δ that will produce a transmission loss of 0.5 dB.

7.9 A lab technician measures the optical power transmitted through a long spool of fiber, and then cuts off 2.2 km of fiber from the spool to measure the attenuation coefficient. If the two measurements yield powers of 3 mW and 10 mW, what is the attenuation coefficient, expressed both in cm^{-1} and in dB/km?

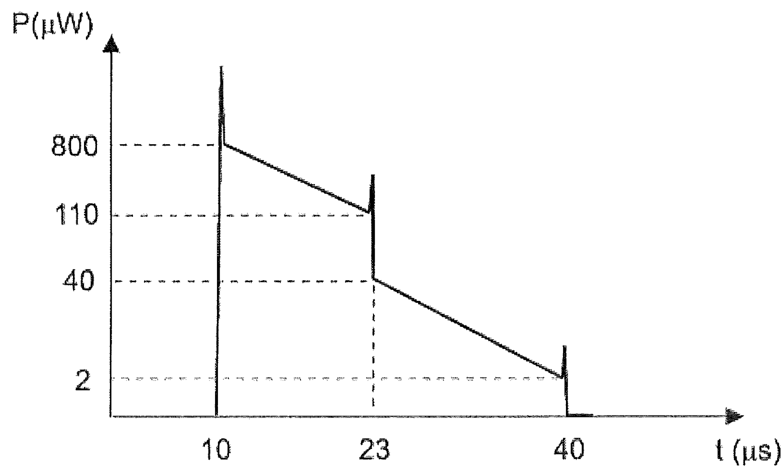

Figure 7-11 Reflected power measured in OTDR for Problem 7.10. The vertical axis is log scale; the horizontal axis is linear.

7.10 A field technician is diagnosing the losses in a fiber optic link consisting of two fibers, A and B, connected together with a mechanical splice. She sends in 800 nm light at the accessible end of fiber A, and looks at the reflected light signal from that same port using an OTDR. The measured time dependence is shown in Fig. 7-11. From this data, how long is the fiber link, and where along the fiber link is the splice located? Assume an index of 1.5.

7.11 For the fiber link of Problem 7.10, what is the one-way transmission loss through the splice, expressed in dB? Also give the reflected power from the splice expressed in dB relative to the power incident on the splice, assuming that the splice loss is due to reflection (rather than, e.g., absorption or scattering).

7.12 For the fiber link of Problem 7.10, determine the attenuation coefficients for the two fibers A and B. Express your answer both in cm^{-1} and in dB/km.

12.3 A large-core, multimode step- index fiber of radius a is excited by a surface-emitting LED with area $A_s < \pi a^2$. The fiber has NA = 0.2 and loss coefficient 4 dB/km. The LEDs total output power is 5 mW. Compute the power propagating in the fiber core at 1 m, 1 km, and 10 km. Repeat this problem if the fiber has instead NA = 0.5 and a loss of 20 dB/km.

12.4 A source has a half-power emission angle of 32.8° as measured from the normal to the emitting surface. Compute the coupling efficiency into a multimode SI fiber having NA = 0.2.

12.5 For a Lambertian emitter, calculate the angle with respect to the surface normal at which the emitted intensity is (a) 50% of the peak intensity, (b) 20% of the peak intensity, and (c) 5% of the peak intensity. (d) What is the full width at half maximum (FWHM) of the Lambertian radiation pattern?

12.10 Consider a surface that emits light with a "flat-top" brightness distribution, such that $B(\theta) = B_0$ for $\theta < \theta_0$ (with B_0 a constant), and $B(\theta) = 0$ for $\theta > \theta_0$. Determine the efficiency with which light from this surface is coupled into a multimode fiber having core radius a, core index n_1, and numerical aperture NA. Write your results in general form, and also in an approximate form valid when NA $\ll 1$ and $\theta_0 \ll 1$.

Study Guide 4

7.2 For $x \ll 1$, $\cos^{-1}(x) \approx \pi/2 - x$, and Eq.(7-1) becomes

$$\eta \approx \frac{1}{\pi}\left\{2\left[\frac{\pi}{2} - \frac{\delta}{2a}\right] - \frac{\delta}{a}\right\} = \boxed{1 - \frac{2\delta}{\pi a}}$$

7.3 Substituting in Eq.(7-1) with $\delta = 1.5\ \mu$m and $a = 25\ \mu$m gives

$$\eta = \frac{1}{\pi}\left\{2\cos^{-1}\left(\frac{1.5}{50}\right) - \frac{1.5}{25}\sqrt{1-\left(\frac{1.5}{50}\right)^2}\right\} = \boxed{0.9618}$$

$1-\eta = .0382$

Using the approximate expression from Problem 7.2 gives

$$\eta \approx 1 - \frac{2\delta}{\pi a} = \frac{(2)(1.5)}{\pi(25)} = 0.9618$$

The two results agree very well in this case since $\delta/a \ll 1$.

7.7 The coupling efficiency corresponding to a 0.2 (0.5) dB loss is

$$\eta = 10^{-0.5/10} = 0.8912 = e^{-(\delta/w)^2}$$

Solving for the transverse displacement gives

$$(\delta/w)^2 = \ln\left(\frac{1}{0.8912}\right) = 0.1151$$

$$\frac{\delta}{w} = 0.339$$

and so $\delta = (0.339)(4.25\ \mu m) = \boxed{1.44\ \mu\text{m}}$

7.9 Using Beer's law,

$$3\text{ mW} = 10\text{ mW}\ e^{-\alpha L}$$

where $L = 2.2$ km $= 2.2 \times 10^5$ cm is the length of fiber cut off. Solving for the attenuation coefficient α gives

$$\alpha = \frac{\ln(10/3)}{2.2 \times 10^5\text{ cm}} = \boxed{5.47 \times 10^{-6}\text{ cm}^{-1}}$$

Converting units, we have

$$\alpha = (5.47 \times 10^{-6}\text{ cm}^{-1})\left(4.34 \times 10^5\,\frac{\text{dB/km}}{\text{cm}^{-1}}\right) = \boxed{2.37\text{ dB/km}}$$

As a check, we can calculate the total attenuation in dB, (2.37 dB/km)(2.2 km) = 5.22 dB, and then the fraction transmitted $T = 10^{-\text{dB}/10} = 0.300$.

7.10 Using Eq.(7-10), the length of the fiber link is

$$L = \frac{c}{2n}t = \frac{3 \times 10^8}{2(1.5)}(30 \times 10^{-6}) = \boxed{3\text{km}}$$

and the location of the splice from the beginning of fiber A is

$$z_{splice} = \frac{c}{2n}t_{splice} = \frac{3 \times 10^8}{2(1.5)}(13 \times 10^{-6}) = \boxed{1.3\text{km}}$$

7.11 Using Eq.(7-11), the one-way transmission efficiency through the splice is $T = \sqrt{(40/110)} = 0.603$. This corresponds to a dB drop of $10\log_{10}(1/0.603) = \boxed{2.2 \text{ dB}}$ in one pass through the splice. The fraction of light reflected from the slice is $1 - T = 0.397$, and this reflected power is down from the power incident on the splice by $10\log_{10}(0.397) = \boxed{-4 \text{ dB}}$ dB.

7.12 The dB loss for light propagating down and back along fiber A is

$$\text{dB loss} = 10\log_{10}\left(\frac{800}{110}\right) = 8.62 \text{ dB}$$

Since the total propagation distance is 2×1.3 km, the attenuation coefficient is

$$\alpha = \frac{8.62 \text{ dB}}{2.6 \text{ km}} = \boxed{3.31 \text{ dB/km}}$$

For fiber B, the propagation distance is 2×1.7 km, and the dB drop is

$$\text{dB loss} = 10\log_{10}\left(\frac{40}{2}\right) = 13.0 \text{ dB}$$

so the attenuation coefficient is

$$\alpha = \frac{13 \text{ dB}}{3.4 \text{ km}} = \boxed{3.83 \text{ dB/km}}$$

An alternative approach is to use Eq.(7-15),

$$\alpha = \frac{n}{ct}\ln\left(\frac{S(0)}{S(z)}\right)$$

For fiber A, this becomes

$$\alpha = \frac{1.5}{(3 \times 10^8)(13 \times 10^{-6})}\ln\left(\frac{800}{110}\right) = 7.63 \times 10^{-4} \text{ m}^{-1} = 7.63 \times 10^{-6} \text{ cm}^{-1}$$

This can be converted into dB/km using Eq.(5-4), with the result $\alpha = 3.31$ dB/km. A similar calculation for fiber B gives $\alpha = 3.83$ dB/km, as before.

12.3 The fraction of LED light that is coupled into the fiber core is $\eta_c = \text{NA}^2 = (0.2)^2 = 0.04$, so the power coupled in is $(5)(0.04) = 0.2$ mW. After propagating a distance L, the power in the core is reduced by the factor $10^{-(\text{dB loss})/10}$. For example, when $L = 1$ km, the dB loss is 4 dB, and the power is reduced by the factor $10^{-(4)/10} = 0.398$. The optical power is then $(0.2)(0.398) = \boxed{0.0796 \text{ mW}}$. Repeating this calculation for $L = 1$ m gives $P = \boxed{0.1998 \text{ mW}}$, and for $L = 10$ km gives $P = \boxed{2 \times 10^{-5} \text{ mW}}$.

For a fiber with NA $= 0.5$ and loss of 20 dB/km, the powers are $\boxed{1.244,\ 0.0125, \text{ and } 1.25\times 10^{-20} \text{ mW}}$ for fiber lengths 1 m, 1 km, and 10 km, respectively.

12.4 The value of m is obtained by setting $0.5 = \cos^m(32.8°)$ and taking the log, which yields

$$\underline{m} = \frac{\ln(0.5)}{\ln(\cos 32.8°)} = \underline{4}$$

Substituting into Eq.(12-18) gives

$$\eta_c \simeq \frac{m+1}{2}\text{NA}^2 = \frac{4+1}{2}(0.2)^2 = \boxed{0.10}$$

12.5 (a) At 50% peak intensity, the angle is given by $\cos\theta = 0.5$, so $\boxed{\theta = 60°}$ = 1.047 rad

(b) At the 20% point, $\cos\theta = 0.2$, so $\theta = \boxed{78.5°}$ = 1.37 rad

(c) At the 5% point, $\cos\theta = 0.05$, so $\theta = \boxed{87.1°}$ = 1.52 rad

(d) The full width is twice the half-width $\theta = 60°$, or FWHM $= \boxed{120°}$ = 2.094 rad

12.10 Following in the manner of Eqs.(12-7) and (12-8), but using the flat-top distribution for $B(\theta)$, we have for the power emitted by the source

$$\begin{aligned} P_0 &= A_s B_0 \int_0^{\theta_0} 2\pi \sin\theta\, d\theta \\ &= -2\pi A_s B_0 \cos\theta|_0^{\theta_0} \\ &= 2\pi A_s B_0 (1 - \cos\theta_0) \end{aligned}$$

The power coupled into the fiber is calculated in the same way, but using an upper limit α_{max} for θ, where $\sin\alpha_{max} = \text{NA}$, and NA is the fiber numerical aperture. Therefore,

$$P_{in} = 2\pi A_s B_0 (1 - \cos\alpha_{max})$$

and the coupling efficiency is

$$\eta_c = \frac{P_{in}}{P_0} = \frac{1 - \cos\alpha_{max}}{1 - \cos\theta_0}$$

where we assume $\alpha_{max} < \theta_0$ (a reasonable assumption).

In the limit of small NA,

$$\cos\alpha_{max} = \sqrt{1 - \sin^2\alpha_{max}} = \sqrt{1 - \text{NA}^2} \approx 1 - \frac{1}{2}\text{NA}^2$$

and so

$$\boxed{\eta_c \approx \frac{\text{NA}^2}{2(1 - \cos\theta_0)}}$$

If, in addition, $\theta_0 \ll 1$ (for laser light, for example), we can expand $\cos x \approx 1 - x^2/2$ to obtain

$$\boxed{\eta_c \approx \frac{\text{NA}^2}{\theta_0^2}}$$

This formula applies only when NA $< \theta_0$. For NA $> \theta_0$, the coupling efficiency is unity.