Problems Study Guide 3

5.2 Light with wavelength 1.3 μm is coupled into a long silica fiber. (a) Determine the attenuation coefficient in units of cm^{-1}, assuming that Rayleigh scattering is the predominant loss mechanism. (b) If the optical power at the beginning of the fiber is 5 mW, determine the optical power at a distance 2.5 km down the fiber. (c) Determine the power a distance 25 km down the fiber.

5.3 In a certain (nonsilica) fiber, the loss due to Rayleigh scattering is 6 dB/km at $\lambda =$ 800 nm. What would be the corresponding Rayleigh scattering loss at $\lambda = 600$ nm?

5.6 Incident light of wavelength 1064 nm is Raman scattered in a glass with localized vibrational frequency $f_v = 20$ THz. Determine the wavelength of the scattered light.

5.9 Argon laser light at 514.5 nm is incident on a gas cell with some unknown molecules, and Raman scattering is observed at 579 nm. Are the unknown molecules CO_2 or CO? (See Problems 5.7 and 5.8 for data.)

5.12 Determine the bend radius at which bending losses become significant, for (a) multimode fiber with NA = 0.25 and core diameter 50 μm, and (b) single-mode fiber with NA = 0.18 and core diameter 8 μm. Assume core index = 1.5.

6.1 A light pulse with wavelength 850 nm passes through a single-mode silica fiber. (a) Determine the time spread of the pulse per unit length due to material dispersion if the spectral width is 20 nm. (b) Repeat if the spectral width is 2 nm. (From Fig. 6-2, take $d^2n/d\lambda^2 = 3 \times 10^{10}\ m^{-2}$.)

6.2 A light pulse with wavelength 1550 nm passes through a single-mode silica fiber. Using Fig. 6-3, determine the time spread of the pulse per unit length due to material dispersion if the spectral width is 2 nm.

6.3 Determine the maximum bit rate for digital modulation using the results in Problems 6.1 and 6.2 for fiber lengths of 100 m and 10 km.

6.4 Pulses with a 2 nm spectral width at two discrete wavelengths 850 and 860 nm are coupled simultaneously into a long step-index, single-mode fiber with core radius 4 μm. (a) Which of these pulses reaches the far end of a 5 km long fiber first? (b) What is the time delay between the two pulses at the fiber end?

6.7 Using Fig. 6-3, estimate the total chromatic dispersion at 1550 nm for a step-index silica fiber having core radius 3 μm. Repeat for a core radius 2 μm.

6.10 Chromatic dispersion in a long fiber link can be "undone" by periodically passing the dispersed pulses through a special dispersion compensation fiber (DCF), which has a chromatic dispersion coefficient of opposite sign. If the DCF has $D_c = -300$ ps/(nm km) at the operating wavelength of 1550 nm, and a length of 1 km, determine the distance required between the DCF insertions along the fiber link. Assume that the fiber link uses standard silica telecommunications fiber of core radius 4 μm.

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5.2 (a) Using Eq.(5-6) with $\lambda = 1.3\ \mu$m, the attenuation coefficient is

$$\alpha_R \simeq (0.8)\left(\frac{1\,\mu\text{m}}{1.3\,\mu\text{m}}\right)^4 = \underline{0.280\ \text{dB/km}}$$

or
$$\left(.28\ \frac{\text{dB}}{\text{km}}\right)(2.5\ \text{km}) = 0.7\ \text{dB}$$
$$10^{-\frac{\text{dB}}{10}} = .851$$
$$P = (5)(.851) = 4.25\ \text{mW}$$

Converting units using Eq.(5-4),

inverted

$$\alpha = (0.28\ \text{dB/km})\left(2.303\times10^{-6}\frac{\text{dB/km}}{\text{cm}^{-1}}\right) = \boxed{6.45\times10^{-7}\ \text{cm}^{-1}}$$

(b) At $L = 2.5$ km $= 2.5\times10^5$ cm,

$$P = P_0 e^{-\alpha L} = (5\ \text{mW})e^{-(6.45\times10^{-7})(2.5\times10^5)} = \boxed{4.25\ \text{mW}}$$

(c)Repeating the above with $L = 25$ km gives $P = \boxed{1.00\ \text{mW}}$.

5.3 Since $\alpha_R \propto \lambda^{-4}$, the ratio of attenuation constants at the two wavelengths is

$$\frac{\alpha_{600}}{\alpha_{800}} = \left(\frac{800}{600}\right)^4 = 3.16$$

Therefore, $\alpha_{600} = (3.16)(6\ \text{dB/km}) = \boxed{18.96\ \text{dB/km}}$.

5.6 The frequency of the incident light is

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{1064\times10^{-9}} = 2.8195\times10^{14}\ \text{Hz} \simeq \underline{282\ \text{THz}}$$

The frequency of the scattered light is $\nu' = \nu - f_a = 282 - 20 = \underline{262\ \text{THz}}$. Converting this to a wavelength gives $\lambda' = c/\nu' = (3\times10^8)/(2.62\times10^{14}) = 1.145\times10^{-6}$ m $= \boxed{1145\ \text{nm}}$

5.9 The incident light has frequency $\nu = c/\lambda = (3\times10^8)/(514.5\times10^{-9}) = 5.83\times10^{14}$ Hz $=$ 583 THz, and the scattered light has frequency $\nu' = c/\lambda' = (3\times10^8)/(579\times10^{-9}) =$ 5.18×10^{14} Hz $=$ 518 THz. The frequency shift is $f_v = \nu - \nu' = 583 - 518 = \boxed{65\ \text{THz}}$, and so these are $\boxed{\text{CO molecules.}}$

5.12 (a) Using NA $\simeq n\sqrt{2\Delta}$ from Eq.(4-5),

$$\Delta \simeq \frac{1}{2}\left(\frac{\text{NA}}{n}\right)^2 = \frac{1}{2}\left(\frac{0.25}{1.5}\right)^2 = \underline{0.0139}$$

Then $R < a/\Delta = (25\,\mu\text{m})/(0.0139) = \boxed{1.8\ \text{mm.}}$

(b) Now

$$\Delta \simeq \frac{1}{2}\left(\frac{0.18}{1.5}\right)^2 = \underline{0.0072}$$

and $R < (4\,\mu\text{m})/(0.0072) = \boxed{0.56\ \text{mm.}}$

6.1 (a) Using Eq.(6-10),

$$\frac{|\Delta t|}{L} = \frac{\lambda}{c}\frac{d^2n}{d\lambda^2}\Delta\lambda = \frac{850 \times 10^{-9}}{3 \times 10^8}(3 \times 10^{10})(20 \times 10^{-9}) = 1.7 \times 10^{-12} \text{ s/m} = \boxed{1.7 \text{ ns/km}} \quad = 1{,}7 \cdot 10^{-12} \frac{s}{m}$$

(b) For $\Delta\lambda = 2$ nm, $\Delta t/L = \boxed{0.17 \text{ ns/km.}}$

6.2 From Fig. 6-3, the material dispersion coefficient at 1550 nm is $D_m \approx 21$ ps/(nm·km). The time spread per unit length is therefore

$$\frac{|\Delta t|}{L} = D_m \Delta\lambda = \left(21 \frac{\text{ps}}{\text{nm} \cdot \text{km}}\right)(2 \text{ nm}) = \boxed{42 \text{ ps/km}} \quad = 4{,}2 \cdot 10^{-14} \frac{s}{m}$$

6.3 Using $\text{BR}_{max} = 1/(2\Delta t)$ from Eq.(3-37), and the values $\Delta t/L$ calculated in the preceeding two problems, we obtain the following values for BR_{max}:

λ(nm)	$\Delta\lambda$(nm)	L(km)	Δt	BR_{max}
850	20	0.1	170 ps	2.94 GHz
850	2	0.1	17 ps	29.4 GHz
1550	2	0.1	4.2 ps	119 GHz or 125 GHz
850	20	10	17 ns	29.4 MHz
850	2	10	1.7 ns	294 MHz
1550	2	10	0.42 ns	1.19 GHz

6.4 (a) For both wavelengths $\lambda_1 = 850$ and $\lambda_2 = 860$ nm, $d^2n/d\lambda^2 > 0$. Defining $\Delta\lambda = \lambda_2 - \lambda_1 = +10$ nm, we see from Eq.(6-9) that $\Delta t = t_2 - t_1 < 0$. This means that longer wavelengths take less time to go a distance L than shorter wavelengths. Therefore, the $\boxed{\text{860 nm pulse arrives at the end first.}}$

(b) The magnitude of the time delay is given by

$$|\Delta t| = \frac{\lambda}{c}\frac{d^2n}{d\lambda^2}L\,\Delta\lambda = \frac{850 \times 10^{-9}}{3 \times 10^8}(3\times10^{10})(5\times10^3)(10\times10^{-9}) = 4.25\times10^{-9} \text{ s} = \boxed{4.25 \text{ ns}}$$

6.7 From the graph in Fig. 6-3 at 1550 nm, we estimate $D_c \approx 15$, $D_m \approx 21$, and $D_w \approx -6$ ps/(nm·km). Since the waveguide dispersion is for a core radius $a = 4$ μm, and $D_w \propto 1/a^2$, we can estimate the waveguide dispersion for other core radii using a simple proportionality. For $a = 3$ μm, $D_w = (4/3)^2(-6) = -10.7$ ps/(nm·km). Since the material dispersion coefficient is independent of core radius (it depends only on the type of glass), the chromatic dispersion coefficient becomes $D_c = 21 - 10.7 = \boxed{10.3 \text{ ps/(nm·km).}}$ Repeating this for $a = 2$ μm, we have $D_w = (4/2)^2(-6) = -24$ ps/(nm·km), which leads to $D_c = 21 - 24 = \boxed{-3 \text{ ps/(nm·km).}}$

6.10 From Fig. 6-3, we estimate the total chromatic dispersion of the communications fiber to be $D_c \simeq 15$ ps/(nm·km). For the total dispersion to be zero after propagating through a length L of the communications fiber and a 1 km section of the DCF,

$$\left(15 \frac{\text{ps}}{\text{nm} \cdot \text{km}}\right) L - \left(300 \frac{\text{ps}}{\text{nm} \cdot \text{km}}\right)(1 \text{ km}) = 0$$

Solving this gives $L = 300/15 = \boxed{20 \text{ km.}}$