Study Guide 2

2.15 Let the lens of focal length f be placed between the filament and the fiber end so that the distance from filament to lens is s_1 (object distance) and the distance from lens to fiber end is s_2 (image distance). We need to find two unknowns: s_1 and f. The distance s_2 is known once s_1 is known, since it is given that $s_1 + s_2 = 20$ cm. There are two conditions on s_1 and s_2. First, the ratio must be

$$\frac{s_2}{s_1} = \frac{h_2}{h_1} = \frac{0.05}{2} = 0.025$$

so that the image of the filament just fits in the core. Substituting into $s_1 + s_2 = 20$ we have

$$s_1 + 0.025 s_1 = 20$$

$$s_1 = \frac{20}{1.025} = 19.51 \text{ cm}$$

Therefore, the lens-fiber distance is $s_2 = 0.488$ cm. The second condition on s_1 and s_2 is that they must obey the lens equation, so

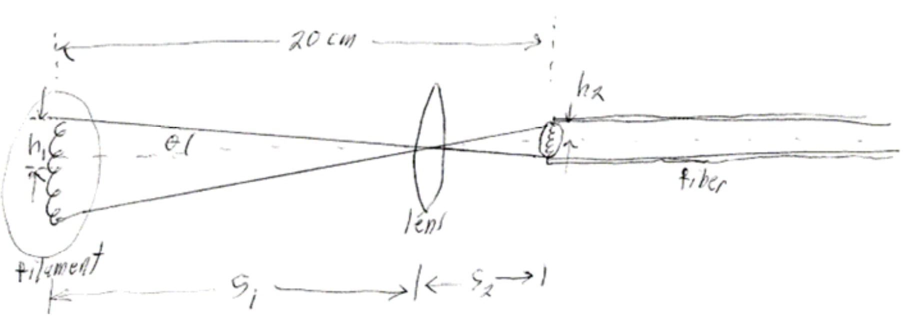
$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{19.51} + \frac{1}{0.488} = 2.101 \text{ cm}^{-1}$$

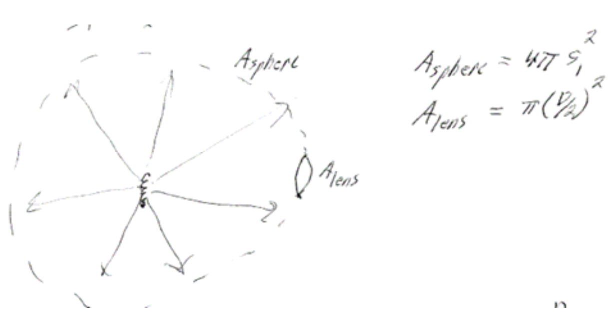
The focal length of the lens is thus $f = 0.476$ cm.

Taking the upper limit on the lens diameter to be $D \approx 0.5$ cm, the fraction of light collected by the lens is the ratio of the lens area to the area of a sphere of radius s_1 centered at the filament. This fraction is evaluated to be

$$\text{frac} = \frac{\pi(D/2)^2}{4\pi s_1^2} = \frac{D^2}{16 s_1^2} = \frac{(0.5)^2}{16(19.5)^2} = 4.1 \times 10^{-5}$$

This very low collection efficiency is the reason that incandescent light sources are not commonly used to couple light into an optical fiber.





3.3 The Vee parameter for the planar waveguide is

$$V_p = \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi(5\,\mu\text{m})}{1.3\,\mu\text{m}}\sqrt{(3.6)^2 - (3.4)^2} = 28.59$$

The number of modes (not including polarization) is therefore int $(V_p/\pi)+1 = \text{int}(9.1)+1 = 10$.

3.4 Eq.(3-24) can be used to solve for the propagation angle θ_m, but it only gives a very approximate result, since it assumes that $m \gg 1$, whereas $m = 2$ here. Using this equation gives

$$\cos\theta_m \simeq \frac{m\lambda_0}{2n_1 d} = \frac{2(1.3\,\mu\text{m})}{2(3.6)(5\,\mu\text{m})} = 0.0722$$

The angle is then $\theta_m \approx 1.5$ rad $= 85.86°$.

The exact result can be obtained by combining Eqs.(3-28) and (3-29) and writing the unknown angle in terms of $x \equiv \cos\theta$. This gives

$$d_m = \frac{\lambda_0}{n_1 x}\left[\frac{m}{2} + \frac{1}{\pi}\cos^{-1}\left(\frac{n_1 x}{\sqrt{n_1^2 - n_2^2}}\right)\right]$$

Substituting the given parameters, this becomes

$$\frac{(5\,\mu\text{m})(3.6)}{1.3\,\mu\text{m}}x = \frac{2}{2} + \frac{1}{\pi}\cos^{-1}\left(\frac{3.6x}{\sqrt{(3.6)^2 - (3.4)^2}}\right)$$

or

$$13.85x = 1 + \frac{1}{\pi}\cos^{-1}(3.043x)$$

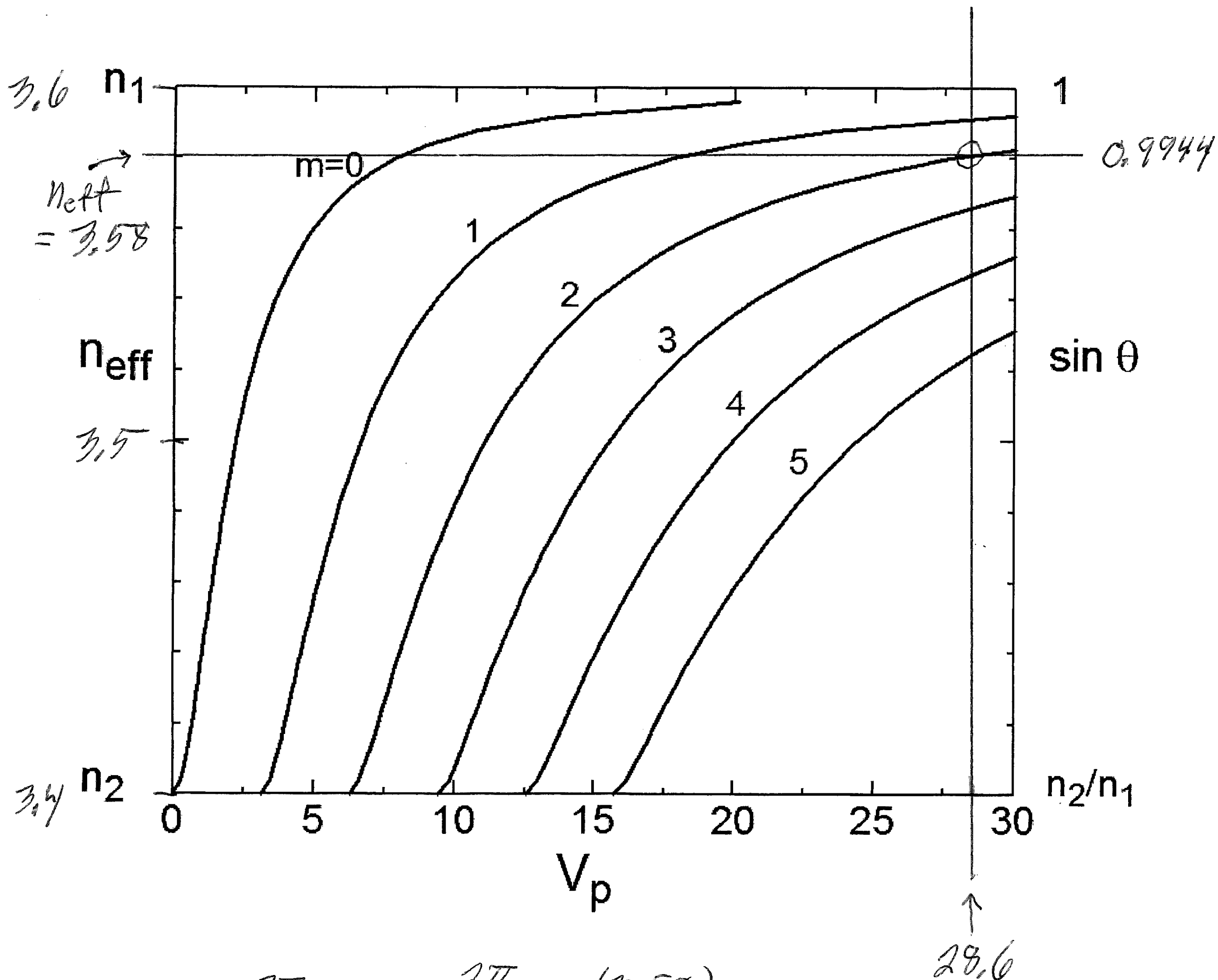
This is an implicit equation for x, which can be solved by evaluating the left and right hand sides for a range of x values, and finding the value of x that makes them equal. The value obtained is $x = 0.101$, which corresponds to a propagation angle $\theta = \cos^{-1}(0.101) = 84.20°$. The effective index is then $n_{eff} = n_1 \sin\theta = 3.6 \sin 84.2° = 3.58$, and the axial propagation constant is

$$\beta = \frac{2\pi n_{eff}}{\lambda_0} = \frac{2\pi(3.58)}{1.3 \times 10^{-6}} = 1.731 \times 10^7 \text{ m}^{-1}$$

Alternatively, the mode chart for planar waveguides can be used for this problem (see next page). First calculate V_p from the given info, and then draw a vertical line on the chart to see where it crosses the $m = 2$ mode line. Drawing a horizontal line through this intersection point, you can determine n_{eff} from where this line crosses the vertical axis. Once n_{eff} is determined, β and θ can be calculated as above.

Prob. 3.4 : Graphical Method

$$V_p = \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi(5\,\mu m)}{1.3\,\mu m}\sqrt{3.6^2 - 3.4^2} = 28.6$$



$$\beta = \frac{2\pi}{\lambda_0} n_{eff} = \frac{2\pi}{1.3\cdot 10^{-6}\,m}(3.58)$$

$$\beta = 1.730 \cdot 10^{7}\ m^{-1}$$

$n_{eff} = n_1 \sin\theta$ so $\sin\theta = \frac{n_{eff}}{n_1} = \frac{3.58}{3.60} = .9944$

$\theta \simeq 84°$

3.8 Eq.(3-26) gives the condition that there be only one mode for $\lambda_{01} = 1.5\ \mu$m. This becomes

$$d < \frac{1.5\,\mu\text{m}}{2\sqrt{(3.6)^2 - (3.4)^2}} = 0.634\,\mu\text{m}$$

Reversing the inequality gives the condition that there be more than one mode for $\lambda_{02} = 1.3\ \mu$m. This becomes

$$d > \frac{1.3\,\mu\text{m}}{2\sqrt{(3.6)^2 - (3.4)^2}} = 0.549\,\mu\text{m}$$

The condition that there be ONLY two modes at $\lambda_{02} = 1.3\ \mu$m (i.e., less than 3 modes) is

$$d < 2\frac{1.3\,\mu\text{m}}{2\sqrt{(3.6)^2 - (3.4)^2}} = 1.098\,\mu\text{m}$$

This last condition is automatically satisfied by the first condition above, and so the desired range of d can be summarized by $0.549 < d < 0.634\ \mu$m.

Here's an alternative approach, based on the V_p parameter:

only 1 mode at 1.5 μm : $V_p = \frac{2\pi d}{\lambda_0} NA < \pi$ — $\lambda_0 = 1.5\,\mu m$

only 2 modes at 1.3 μm : $V_p = \frac{2\pi d}{\lambda_0'} NA < 2\pi$ — $\lambda_0' = 1.3\,\mu m$

more than 1 mode at 1.3 μm : $V_p = \frac{2\pi d}{\lambda_0'} NA > \pi$

so $\frac{\lambda_0'}{2NA} < d < \frac{\lambda_0}{2NA}$

3.9 For the 4th mode (with $m = 3$) to be cut off, we require (see Fig. 3-5)

$$3\left(\frac{\lambda_0}{2n_1 d}\right) > \cos\theta_c$$

This can be written

$$\lambda_0 > \frac{2n_1 d\cos\theta_c}{3} = \frac{2n_1 d\sqrt{1-(n_2/n_1)^2}}{3} = \frac{2}{3}d\sqrt{n_1^2 - n_2^2}$$

or

$$\lambda_0 > \frac{2}{3}(1.2\,\mu\text{m})\sqrt{(3.6)^2 - (3.4)^2} = 0.946\,\mu\text{m}$$

For the 3rd mode (with $m = 2$) NOT to be cut off, we require

$$2\left(\frac{\lambda_0}{2n_1 d}\right) < \cos\theta_c$$

which becomes

$$\lambda_0 < \frac{2n_1 d\sqrt{1-(n_2/n_1)^2}}{2} = d\sqrt{n_1^2 - n_2^2} = (1.2)\sqrt{(3.6)^2 - (3.4)^2} = 1.420\,\mu\text{m}$$

The above two restrictions can be summarized by $0.946 < \lambda_0 < 1.420\ \mu$m.

Another approach for problem 3.9 is to consider how many modes can propagate for a given V_p value (see mode chart):

$V_p < \pi$ — 1 mode

$\pi < V_p < 2\pi$ — 2 modes

$2\pi < V_p < 3\pi$ — 3 modes

To have 3 modes (but not 4), the last condition becomes

$$2\pi < \frac{2\pi d}{\lambda_0} NA < 3\pi \qquad \text{so} \qquad \lambda_0 < 1{,}420\ \mu m$$

$$\lambda_0 < d \cdot NA < \frac{3}{2}\lambda_0 \qquad\qquad \lambda_0 > 0.947\ \mu m$$

$$\lambda_0 < 1{,}420\ \mu m < \frac{3}{2}\lambda_0$$

3.10 From the definition $\Delta \equiv (n_1 - n_2)/n_1$, we have $n_2/n_1 = 1 - \Delta$. The range of angles in Eq.(3-34) then becomes

$$1 - \Delta < \sin\theta < 1$$

$$0.985 < \sin\theta < 1$$

$$80.1^\circ < \theta < 90^\circ$$

If the rays make an angle ψ with the waveguide axis, then $\theta + \psi = 90^\circ$, and the range of angles becomes

$$0 < \psi < 9.9^\circ$$

3.11 Eq.(3-35) gives

$$\Delta t = \frac{Ln}{c}\Delta = \frac{(2.5 \times 10^3)(1.48)(0.015)}{3 \times 10^8} = 1.85 \times 10^{-7}\ \text{s}$$

The maximum bit rate is then

$$BR_{max} = \frac{1}{2\Delta t} = \frac{1}{2(1.85 \times 10^{-7})} = 2.70 \times 10^6\ \text{bits/s}$$

4.3 The V parameter is defined in Eq.(4-9) as

$$V = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi(25 \times 10^{-6})}{1.3 \times 10^{-6}}\sqrt{(1.495)^2 - (1.485)^2} = 20.85$$

The number of modes is then $\approx V^2/2 = (20.85)^2/2 = 218$. This includes different polarizations as different modes.

4.6 The fiber numerical aperture is $\text{NA} \simeq n\sqrt{2\Delta} = 1.48\sqrt{2(0.013)} = 0.239$. We are given the cutoff wavelength $\lambda_c = 1.25\,\mu\text{m}$, and so Eq.(4-14) gives

$$a = \frac{(2.405)\lambda_c}{2\pi\text{NA}} = \frac{(2.405)(1.25\,\mu\text{m})}{2\pi(0.239)} = 2.00\,\mu\text{m}$$

4.7 From the mode chart in Fig. 4-9, we see that if $2.5 < V < 3.7$, there will be 4 curve crossings for a line drawn vertically. This corresponds to 8 modes when different polarizations are considered as different modes. Using the definition $V = 2\pi a\text{NA}/\lambda$ and solving for λ, we have

$$\lambda < \frac{2\pi(2\,\mu\text{m})(0.239)}{2.5} = 1.20\,\mu\text{m}$$

$$\lambda > \frac{2\pi(2\,\mu\text{m})(0.239)}{3.7} = 0.812\,\mu\text{m}$$

The range of wavelengths is then $0.812 < \lambda < 1.20\,\mu\text{m}$

4.10 The Vee parameter for the fiber is

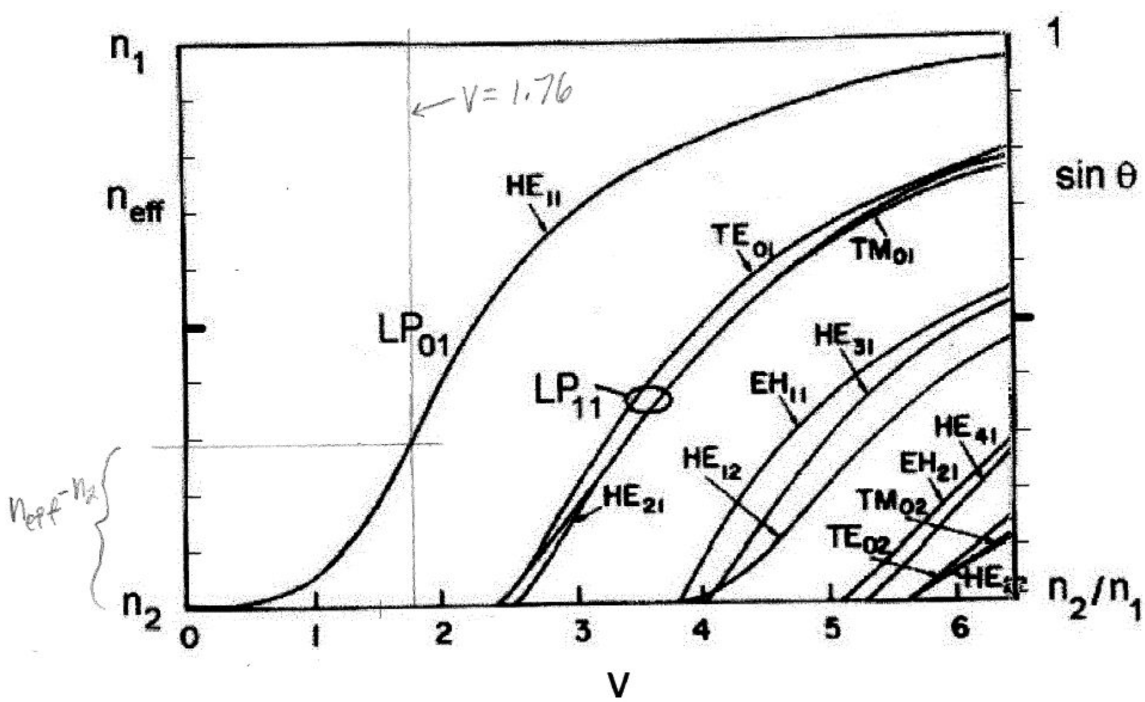
$$V = \frac{2\pi a\text{NA}}{\lambda} = \frac{2\pi(2\,\mu\text{m})(0.239)}{1.7\,\mu\text{m}} = 1.76$$

From Fig. 4-9, a vertical line at $V = 1.76$ intersects the LP_{01} curve at a point where $(n_{eff} - n_2) \approx 0.29(n_1 - n_2)$. The index in the cladding is $n_2 = n_1(1-\Delta) = 1.48(1 - 0.013) = 1.4608$. The effective index is therefore

$$n_{eff} \approx n_2 + 0.29(n_1 - n_2) = 1.4608 + (0.29)(1.48 - 1.4608) = 1.4663$$

From Eq.(4-15) we then have

$$\beta = \frac{2\pi}{\lambda_0}n_{eff} = \frac{2\pi(1.4663)}{1.7\times10^{-6}} = 5.4196\times10^6\ \text{m}^{-1}$$



$n_{eff} - n_2 \simeq 0.29(n_1 - n_2)$

4.11 (a) Using Eq.(4-18) with $V = 1.76$,

$$w \simeq (2\,\mu\text{m})\left[0.65 + \frac{1.619}{(1.76)^{1.5}} + \frac{2.879}{(1.76)^{6}}\right] = 2.88\,\mu\text{m}$$

The mode field diameter is then $2w = 5.76\ \mu$m.

(b) The intensity varies as the square of the E field, and using Eq.(4-17) we have

$$\frac{I}{I_0} = 0.05 = e^{-2r^2/w^2}$$

$$\ln 20 = 2(r/w)^2$$

$$\frac{r}{w} = \sqrt{\frac{\ln 20}{2}} = 1.224$$

$$r = (1.224)(2.88\,\mu\text{m}) = 3.52\,\mu\text{m}$$