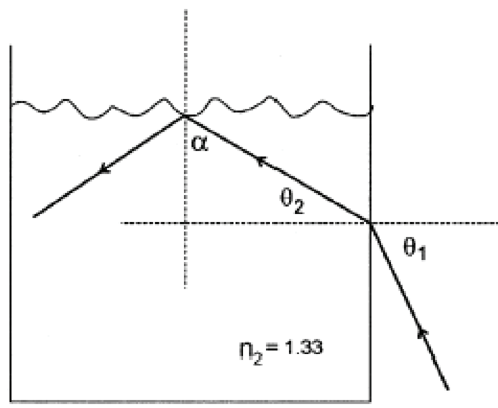
Study Guide 1

2.3 From the diagram below, we have



$$\sin\theta_1 = n\sin\theta_2 = n\cos\alpha = n\sqrt{1-\sin^2\alpha}$$

where $n_1 = 1$ and $n_2 = n = 1.33$. Since TIR requires that $\sin\alpha > 1/n$,

$$\sin\theta_1 < n\sqrt{1-1/n^2} = \sqrt{n^2-1} = 0.877$$

The incident angle must therefore be $\theta_1 < 61.3°$.

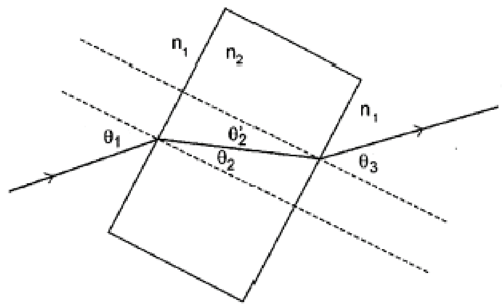
2.4 The beam intensity is

$$I = \frac{P}{A} = \frac{10^{-3}}{\pi(5\times10^{-4})^2} = 1.27\times10^3\ \mathrm{W/m^2}$$

This is related to the peak E field by $I = c\epsilon_0 E^2/2$, assuming $n = 1$, so

$$E = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1.27\times10^3)}{(3\times10^8)(8.85\times10^{-12})}} = 978\ \mathrm{V/m}$$

2.7 Consider the situation shown below, in which Brewster's condition holds for the first interface. It was already shown in problem 2.1 that $\theta_2' = \theta_2$ and $\theta_3 = \theta_1$ if the two faces are parallel. We will take $n_1 = 1$ (air) and $n_2 = n$, so Brewster's condition at



the first interface is $\tan\theta_1 = n$. What we need to show is that Brewster's condition also holds for the second interface, i.e., $\tan\theta_2' = n_3/n_2$ or $\tan\theta_2 = 1/n$. We can do this by combining the first Brewster's condition with Snell's law, to obtain the second Brewster's condition. Since Snell's law involves the sine of the angles, we first put the Brewster's condition in terms of $\sin\theta_1$,

$$\tan\theta_1 = \frac{\sin\theta_1}{\cos\theta_1} = \frac{\sin\theta_1}{\sqrt{1-\sin^2\theta_1}} = n$$

Squaring and rearranging gives

$$\sin^2\theta_1 = n^2\left(1-\sin^2\theta_1\right)$$

$$(n^2+1)\sin^2\theta_1 = n^2$$

$$\sin\theta_1 = \frac{n}{\sqrt{n^2+1}}$$

Now using Snell's law, $\sin\theta_1 = n\sin\theta_2$, we have

$$\sin\theta_2 = \frac{1}{\sqrt{n^2+1}}$$

and

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\frac{1}{n^2+1}} = \frac{n}{\sqrt{n^2+1}}$$

Combining the above two equations then yields

$$\tan\theta_2 = \frac{\sin\theta_2}{\cos\theta_2} = \frac{1}{n}$$

This is the condition for Brewster's angle at the second interface, which was to be proved.

A more concise derivation is to note that at Brewster's angle, $\theta_1 + \theta_2 = 90°$. Sketching a 90° triangle, it is easily seen that if $\tan\theta_1 = n$, then $\tan\theta_2 = 1/n$.

2.8 (a) Using Eq.(2-21), we have

$$\sin^2\theta_1 - \sin^2\theta_c = \left(\frac{\lambda_0}{2\pi n_1 \delta}\right)^2 = \left(\frac{1\times 10^{-6}}{2\pi(1.5)(20\times 10^{-6})}\right)^2 = 2.81\times 10^{-5}$$

Since the difference in angle $\Delta\theta = \theta_1 - \theta_c$ is very small, we can make the approximation

$$\Delta(\sin^2\theta) \approx 2\,\sin\theta\,\cos\theta\,\Delta\theta$$

where $\theta \approx \theta_c = \sin^{-1}(1/1.5) = 41.8°$. Therefore

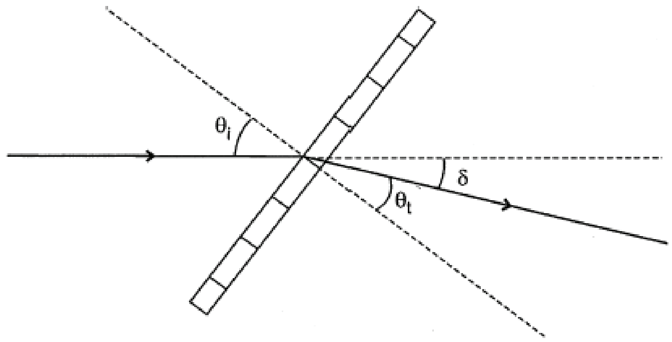
$$\Delta\theta = \frac{2.81\times 10^{-5}}{2\,\sin 41.8\,\cos 41.8} = 2.83\times 10^{-5}\text{ rad} = 1.62\times 10^{-3}\text{ deg}$$

(b) For a beam of diameter D, the approximate divergence angle is $\sim \lambda/D$, where λ is the wavelength in the medium. To have this less than $\Delta\theta$, we would need

$$D > \frac{\lambda}{\Delta\theta} = \frac{1\times 10^{-6}}{(1.5)(2.83\times 10^{-5})} = 2.35\times 10^{-2}\text{ m} = 2.35\text{ cm}$$

2.11 (a) In this case $\theta_i = 0$, so $\sin_t = m\lambda/d = m(0.5/3) = m/6$. For $m = 1$, $\theta_1 = 9.59°$ and for $m = 2$, $\theta_2 = 19.5°$.

(b) As seen in the diagram below, the deflection of the beam from its original direction is $\delta = \theta_i - \theta_t$. The incident and diffracted angles obey



$$d(\sin\theta_i - \sin\theta_t) = m\lambda$$

where the minus sign is due to taking positive θ_i and θ_t on opposite sides of the grating normal. Using $\theta_i = 40°$ and $\lambda/d = 1/6$, we have

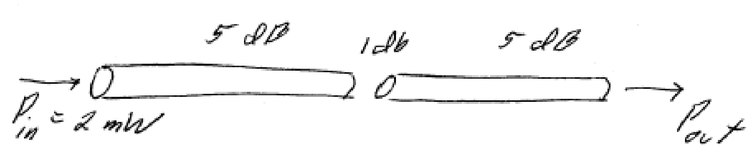
$$\sin\theta_t = 0.643 - m/6$$

The calculated angles θ_t and δ are tabulated below for the lowest two orders.

m	θ_t (deg)	δ (deg)
-2	77.5	-37.5
-1	54.1	-14.1
0	40	0
1	28.4	11.6
2	18	22

It can be seen that the angular deflection is no longer symmetric about the original beam direction, but is instead skewed in the direction away from the grating normal.

SP#1:



$$P_{out} = 10^{-\frac{dB}{10}} P_{in} = 10^{-\frac{5+1+5}{10}} P_{in}$$

$$P_{out} \approx 0.0794\, P_{in} = \boxed{0.159 \text{ mW}}$$

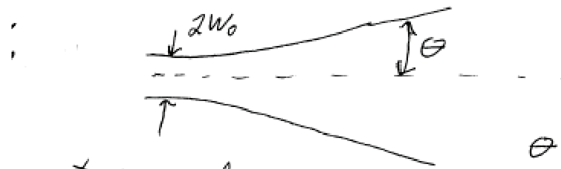
SP#2:

$$\text{Power} = \frac{\text{Energy}}{\text{photon}} \cdot \frac{\text{photons}}{\text{time}}$$

$$P = h\nu \cdot R$$

$$R = \frac{P}{h\nu} = \frac{10^{-9}\,W}{1.5\cdot 10^{-19}\,J} = 6.6\cdot 10^{9}\ s^{-1}$$

SP#3:

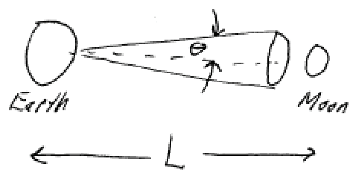


spot size radius $w_0 = 1$ mm

$$\theta = \frac{\lambda}{\pi w_0} = \frac{8\cdot 10^{-7}}{\pi(1\cdot 10^{-3})}$$

$$\theta = \boxed{2.54\cdot 10^{-4}\ \text{rad.}}$$

half-angle



Beam diameter at moon

$$D = 2\theta L = 2(2.54\cdot 10^{-4})(3.8\cdot 10^{8})$$

$$D_{moon} = 1.93\cdot 10^{5}\ m$$

SP#4:

air $\quad n \simeq 1$

GaAs $\quad n \simeq 3.6$

$Al_{0.3}Ga_{0.7}As \quad n \simeq 3.4$

<u>AlGaAs / air</u> $\qquad R \simeq \dfrac{(3.4-1)^2}{(3.4+1)^2} \simeq 0.297$

$$\text{dB loss} = 10 \log_{10}\left(\frac{P_{in}}{0.703\,P_{in}}\right) = \boxed{1.53 \text{ dB}}$$

<u>GaAs / AlGaAs</u>

$$R = \frac{(3.6-3.4)^2}{(3.6+3.4)^2} = 8.16 \cdot 10^{-4}$$

$$\text{dB loss} = 10 \log_{10}\left(\frac{1}{0.99918}\right) = \boxed{0.0035 \text{ dB}}$$

This illustrates the idea of index matching to reduce reflection losses.