PH 2301 Electromagnetic Fields I:

STUDY GUIDE #2

Curvilinear coordinates

You need a good understanding of alternative coordinate systems to the Cartesian. The spherical polar coordinates (§ 1.4.1) and the cylindrical coordinates (§ 1.4.2) are of great importance to Electrostatics. In problems with central symmetry, the spherical coordinates along with corresponding vector derivatives (Eqs 1.70-1.73) are very convenient for use; while the cylindrical coordinates with corresponding vector relations (Eqs 1.79-1.82) is an appealing system to use for a "wire-type" problems.

However, you must be aware of the "poisonous snake" warning on the page 39. Because the unit vectors in curvilinear coordinates are position-dependent you **can not** blindly take these unit vectors outside an integral or a derivative as we do with Cartesian unit vectors.

Electrostatics

The entire Electromagnetic Theory is hidden in 4 Maxwell equations. In this course we are starting with the simplest - static case. This is when *nothing* in the system depends on time. In particular, for a next few weeks we'll assume that charges are *stationary* or *moving with a constant speed.* In that case, 4 Maxwell equations can be split into **two independent** (no time dependence- no effects of E on B and B on E) pairs of equations: a pair describing *electrostatics* and a pair describing *magnetostatics*. From the perspective of a field theory, these two pairs of equations are perfect for studying properties of a vector field. The first pair (from *electrostatics*) deals with a vector field with zero curl and a given non-zero divergence. The second pair (*magnetostatics*) deals with a vector field with a given non-zero curl and zero divergence.

I. Study § 2.1 CAREFULLY.

In principle, you've all learned electromagnetism in the PH 1120/21 course, but that was a relatively superficial overview. In Chapter 2, electrostatics is presented again from the beginning, with some very important differences:

1. Continuous charge distributions are emphasized.
2. Mathematical expressions are more general.
3. Differential and integral versions are given and are more often the most convenient to use.

While studying Chapter 2, be sure to understand completely the material as you read before moving on. Remind yourself of the connections with the material covered in PH 1120/21. DO NOT skip over any "math" that looks intimidating, if you take the time to read it carefully, you'll probably find that it makes sense to you. ALWAYS look for the physical interpretation of any mathematical expression.

You may find yourself referring back to unassigned parts of Chapter 1. That's OK. If you can't follow it, bring your questions to class.

II. Study § 2.2.

A. Try to avoid the phrase "...force *between* two particles". It helps to avoid confusion to keep clearly in mind that the force acts *on* one particle, and is exerted *by* the other. That the third law assures an equal and opposite force exerted by the first particle on the second does not mean there is one force "between" the two. If you become accustomed to writing Coulomb's law in vector form, the vectors help you keep track of which particle is acted on, and in what direction.

B. Standard notation is often confusing. For example, we all use r to indicate the radius of an arc. We also use **r** to indicate the position of a point with respect to the origin of some coordinate system. (Thus, the tendency to call it the "radius vector", when there isn't really a radius involved).

It gets worse. In electrostatics, we often specify the location of charges that are exerting forces by vectors from the origin of a coordinate system to the charges. We also frequently specify the location of a charge being *acted on*, or the location of a field point by a vector. Finally, we might specify the displacement of one charge from another by a vector that is frequently called.

Moral: pay close attention to notation. If there's more than one position vector or displacement vector in your problem, be sure that you distinguish among them.

Griffiths has a non-standard system of notation that can be helpful. Vectors from the origin to charges that are the *source* of a force or a field are given a prime:

Vectors from the origin to a charge that *feels* the force, or to a *field point*, don't have the prime: **r**. The displacement of the field point from the source charge is given a "script **r** designation.

Pay attention to notation. Don't let it intimidate you! Keep scratch paper handy, and be sure you understand what each symbol means. Draw sketches as you go. (E.g., does $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ point *from* position $\mathbf{r}_i$ *to* position $\mathbf{r}_j$ or *vice versa*?) As another example, you will need to keep firmly in mind the distinction between **r** and **r**' when both symbols appear in integrands; be sure you know which is the dummy variable of integration, and why.

C. With Coulomb's law and superposition, all electrical problems are solved "in principle" When you see "in principle", it usually means that it's not very useful!

D. These sections lead you into applications of Gauss's law. Carefully study cases of the special symmetry for which Gauss's Law is tremendously useful (Examples 2.2-2.4). Some (if not all) of these special cases should be familiar to you from PH1120. Also pay special attention to the important conclusion that the **curl** of the electro**static** field is ALWAYS zero.

* The word 'electro**static**" is emphasized because the $\nabla\times\mathbf{E}$ (curl **E**) is NOT zero in a region where there are time-varying fields. Stay tuned!

III. Study carefully ALL of § 2.3.

A. The electric potential is introduced in these sections. Note that the three statements: "**E** is a conservative field", "$\int \mathbf{E} \cdot d\mathbf{l}$ is independent of the path", and "$\nabla \times \mathbf{E} = 0$" are ALL equivalent. Each allows us to define a scalar potential function such that $\mathbf{E} = -\nabla V$.

B. The potential difference between two points, *a* and *b*, is given by

$$V_b - V_a = -\int_a^b \mathbf{E} \bullet d\mathbf{l}$$

The potential at the point *b* is given by

$$V_b = -\int_\infty^b \mathbf{E} \bullet d\mathbf{l}$$

C. Example 6 on page 81 deserves some comment. First, the path integral is all a radial path from the point at infinity to the point at r. Second, in this context the use of the prime in r' does not indicate the coordinate of some charge, but merely designates a "dummy" variable of integration, since the upper limit of the integral is the fixed point r.

Finally, things pass quickly from a scalar product $\mathbf{E} \cdot d\mathbf{l}$ to an integral in dr'. There are some potentially confusing minus signs buried in that transition, so it pays to look at it carefully.

$$\mathbf{E} \bullet d\mathbf{l} = \frac{q}{4\pi\varepsilon_o}\left(\frac{\hat{\mathbf{r}} \bullet d\mathbf{l}}{r^2}\right)$$

$$d\mathbf{l} = dl(-\hat{\mathbf{r}}) = -dr'(-\hat{\mathbf{r}}) = dr'\hat{\mathbf{r}}$$

Here, $dl = -dr'$ since the value of r' decreases as the path length increases. From here, you should be able to follow the rest on page 82.

D. Note the comparison between Equations 2.29 and 2.8, and the special, and forgettable warning, at the bottom of page 89. Study Example 2.7: compare this solution with Probs. 2.7 and 2.11 in which the potential can be found from **E** using Eq. 2.21. The cross-comparison should illustrate the power of the Gauss's law as well as the potential formulation.

E. There's a TON of physics in § 2.3.5! Make sure you work through a suggested Problem 2.30.