

PH2201 – Intermediate Mechanics I
Study Guide 5

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| Readings |
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| <u>Topic</u> | <u>Book/section</u> | <u>Comments</u> |
|--|---|---|
| Central forces and gravity | Kleppner 10.1-10.2 Taylor 4.8 Morin 5.4 | When the force between two particles is directed along the line between the particles, the analysis of motion is greatly simplified. Gravity is an important example. |
| Conservation of angular momentum with central forces | Kleppner 7.2 Taylor 3.4 Morin 7.1 | Using the angular momentum L as a constant of the motion further simplifies central force problems |
| Effective potential | Kleppner 10.3,10.4 Morin 7.2-7.3 Taylor 8.4-8.5 | Study example 10.3 The complete solution for central force problems can be obtained by solving a one dimensional problem with an "effective potential", which incorporates the constant angular momentum |

Practice Problems from Kleppner
(not to be turned in for grading)

Problem 10.2 -- except take $m = 0.05$ kg, $F = 40 r^3$ N with r in meters, and $L = 10^{-4}$ kg-m²/s

Also add:

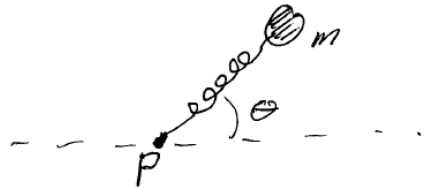
- (d) Find the frequency of small oscillations (in rad/s) in the radial direction for a nearly circular orbit.

Problem 10.6 -- [to simplify the algebra, define a variable $B = L^2/(2m)$]

Homework Problems

Homework set 5 is **due Tuesday Oct. 6:**

1. A mass m is attached to one end of a spring of spring constant k , and the other end of the spring is attached to a fixed pivot point P . Assume that the rest length of the spring is negligible, so the restoring force has magnitude kr , where r is the distance from point P to the mass. You give the mass a sideways push so it whorls around the pivot with some angular momentum L .



 - a) Using the effective potential approach, determine the equilibrium position r_0 such that the mass moves in a circle around point P .
 - b) Determine the angular speed of rotation about P , $\omega_{\text{rot}} = \dot{\theta}$, for the circular motion of part (a), and express your answer in terms of the oscillation frequency $\omega_0 = \sqrt{k/m}$ for the linear harmonic oscillator.
 - c) The mass is now pulled to position $r = r_0 + A$ and released, while it is spinning around. Find the oscillation frequency ω_{osc} around the equilibrium position r_0 . (assume $A \ll r_0$)
 - d) Determine the orbit equation $r(\theta)$ for part (c).
2. For a given angular momentum L , find the potential energy function $U(r)$ that leads to a spiral path of the form $r = r_0 \theta^k$. Choose the total energy E to be zero. Hint: obtain an expression for \dot{r} that is only a function of r , not θ , and then use the energy equation with effective potential energy.
3. Prove that for circular orbits around a given gravitational force center (such as the sun), the speed of the orbiting body is inversely proportional to the square root of the orbital radius.
4. A particle of mass m moves under a central, repulsive force of magnitude $F = mb/r^3$, and is initially located a distance a from the origin of the force, and moving with a speed v in a direction perpendicular to the line between the particle and the force center. Show that the equation of the path of the particle is given in polar coordinates (r, θ) by

$$r \cos(p\theta) = a$$

where $p^2 = 1 + b/(av)^2$. Also sketch this path, and indicate whether or not this is a closed orbit.

5. A particle of mass m is moving on a frictionless horizontal table and is attached to a massless string, whose other end passes through a hole in the table, where I am holding it. Initially the particle is moving in a circle of radius r_0 with angular velocity ω_0 , but now I pull the string down through the hole until a length r remains between the hole and the particle. (a) What is the particle's angular velocity now? (b) Assuming that I pull the string so slowly that we can approximate the particle's path by a circle of slowly shrinking radius, calculate the work I did in pulling the string. (c) Compare your answer in part b to the particle's gain in kinetic energy.
6. Kleppner problem 10.4 [to simplify the algebra, define a variable $B = L^2/(2m)$]