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PH2201 – Intermediate Mechanics I
Study Guide 5

Readings

Topic	Book/section	Comments
Central forces and gravity	Kleppner 10.1-10.2 Taylor 4.8 Morin 5.4	When the force between two particles is directed along the line between the particles, the analysis of motion is greatly simplified. Gravity is an important example.
Conservation of angular momentum with central forces	Kleppner 7.2 Taylor 3.4 Morin 7.1	Using the angular momentum *L* as a constant of the motion further simplifies central force problems
Effective potential	Kleppner 10.3,10.4 Morin 7.2-7.3 Taylor 8.4-8.5	Study example 10.3 The complete solution for central force problems can be obtained by solving a one dimensional problem with an "effective potential", which incorporates the constant angular momentum

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Practice Problems from Kleppner

(not to be turned in for grading)

Problem 10.2 -- except take $m = 0.05$ kg, $F = 40\, r^3$ N with r in meters, and $L = 10^{-4}$ kg-m^2/s
Also add:
(d) Find the frequency of small oscillations (in rad/s) in the radial direction for a nearly circular orbit.

Problem 10.6 -- [to simplify the algebra, define a variable $B = L^2/(2m)$]

Homework Problems

Homework set 5 is **due Tuesday Oct. 6:**

1. A mass m is attached to one end of a spring of spring constant k, and the other end of the spring is attached to a fixed pivot point P. Assume that the rest length of the spring is negligible, so the restoring force has magnitude kr, where r is the distance from point P to the mass. You give the mass a sideways push so it whorls around the pivot with some angular momentum L.
   a) Using the effective potential approach, determine the equilibrium position r_0 such that the mass moves in a circle around point P.
   b) Determine the angular speed of rotation about P, $\omega_{\text{rot}} = \dot{\theta}$, for the circular motion of part (a), and express your answer in terms of the oscillation frequency $\omega_0 = \sqrt{k/m}$ for the linear harmonic oscillator.
   c) The mass is now pulled to position $r = r_0 + A$ and released, while it is spinning around. Find the oscillation frequency ω_{osc} around the equilibrium position r_0. (assume $A \ll r_0$)
   d) Determine the orbit equation $r(\theta)$ for part (c).

2. For a given angular momentum L, find the potential energy function $U(r)$ that leads to a spiral path of the form $r = r_0\theta^k$. Choose the total energy E to be zero. Hint: obtain an expression for $\dot{r}$ that is only a function of r, not θ, and then use the energy equation with effective potential energy.

3. Prove that for circular orbits around a given gravitational force center (such as the sun), the speed of the orbiting body is inversely proportional to the square root of the orbital radius.

4. A particle of mass m moves under a central, repulsive force of magnitude $F = mb/r^3$, and is initially located a distance a from the origin of the force, and moving with a speed v in a direction perpendicular to the line between the particle and the force center. Show that the equation of the path of the particle is given in polar coordinates (r,θ) by

$$r\cos(p\theta) = a$$

   where $p^2 = 1 + b/(av)^2$. Also sketch this path, and indicate whether or not this is a closed orbit.

5. A particle of mass m is moving on a frictionless horizontal table and is attached to a massless string, whose other end passes through a hole in the table, where I am holding it. Initially the particle is moving in a circle of radius r_0 with angular velocity ω_0, but now I pull the string down through the hole until a length r remains between the hole and the particle. (a) What is the particle's angular velocity now? (b) Assuming that I pull the string so slowly that we can approximate the particle's path by a circle of slowly shrinking radius, calculate the work I did in pulling the string. (c) Compare your answer in part b to the particle's gain in kinetic energy.

6. Kleppner problem 10.4 [to simplify the algebra, define a variable $B = L^2/(2m)$]