

**PH2201 – Intermediate Mechanics I**  
**Study Guide 4**

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**Readings**

<b><u>Topic</u></b>	<b><u>Book/section</u></b>	<b><u>Comments</u></b>
Work-energy theorem: 1-D and 3-D	Kleppner 5.1-5.3 Taylor 4.6 Morin 5.1	We will use $U$ for potential energy. Note that some authors (e.g. Taylor) use $V$
Conservative forces Potential energy Conservation of mechanical energy	Kleppner 5.4-5.6 Morin 5.3 Taylor 4.1-4.5 Kleppner Note 2	study examples 5.2, 5.3, 5.5, 5.12, 5.14, 5.16  Read the discussion of the gradient and curl in Kleppner pp. 200-205. The curl of a conservative force is zero everywhere.
Energy diagrams in 1-D Small oscillations about potential energy minima	Kleppner 5.7 Kleppner 6.1-6.3 Taylor 5.1 Morin 5.2	Study section 5.7 carefully. Study examples 6.1, 6.2, 6.4 For problems, you need to know the relation between $U(x)$ and the effective “spring constant” $k$ . Make sure you get this essential point.

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**Practice Problems**

Solutions provided in Morin -- **not to be turned in for grading**.

Morin Chapter 5: problems 3, 4, 5, 8, 9, 13

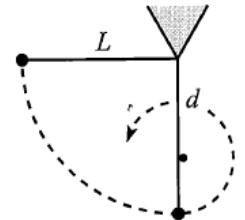
## Homework Problems

Homework set 4 is due Friday Sept. 25:

1. Kleppner 5.6 except let the block have initial speed  $v_0$  at the top. Also add part (b): for what range of initial speeds  $v_0$  does the block immediately lose contact with the sphere? Explain why your answer to part (b) is physically plausible. [compare your result for  $v_0=0$  with book's answer]

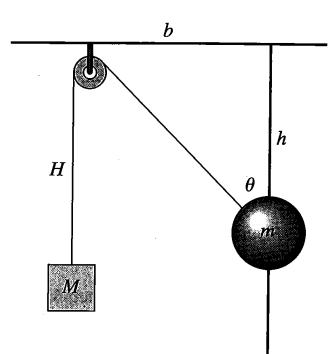
2. A pendulum of length  $L$  is held with its string horizontal, and then released. The string runs into a peg a distance  $d$  below the pivot, as shown in the figure. What is the smallest value of  $d$  for which the string remains taut at all times?

3. A fixed hollow frictionless cone is positioned with its tip pointing down. A particle is released from rest on the inside surface. After it has slid part way down to the tip, it bounces elastically off a platform, which is angled such that the particle ends up being deflected horizontally along the surface (i.e., perpendicular to the paper in the diagram). If the resulting motion of the particle is a horizontal circle around the cone, what is the ratio of the initial height of the particle to the height of the platform? [Note: the heights are measured vertically from the tip of the cone]



4. Kleppner 6.2 [also sketch  $U(x)$  and obtain an expression for  $x_0$ ]

5. A metal ball of mass  $m$  with a hole through the center is threaded on a frictionless vertical rod. A massless string of length  $l$  is attached to the ball, and runs over a massless, frictionless pulley, supporting a block of mass  $M$ . The positions of the two masses can be specified by the one angle  $\theta$ . (a) Write down the potential energy  $U(\theta)$  in terms of the constants  $b$ ,  $l$ ,  $m$ , and  $M$  (assume that the ball and pulley have negligible size). (b) By differentiating  $U(\theta)$ , find whether the system has an equilibrium position, and for what values of  $m$  and  $M$  equilibrium can occur. Discuss the stability of any equilibrium positions.



6. A two-dimensional force  $\mathbf{F} = (x^2, 2xy)$  acts on a particle of mass  $m$  as it moves from the origin to point P at  $(1,1)$ .

- Find the work done by the force if the path of the particle consists of two straight-line segments, the first from  $(0,0)$  to  $(0,1)$ , and the second from  $(0,1)$  to  $(1,1)$ .
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- Compare your answers to parts a and b, and determine whether the force is conservative. Check this by taking the curl of  $\mathbf{F}$ .

7. A bead of mass  $m$  slides without friction on a wire that is bent into the shape described by the function  $y = ae^{b|x|}$ , where  $a$  and  $b$  are constants. The wire is oriented so that  $x$  and  $y$  are in the vertical plane, with the  $y$  axis pointing upward. The bead is initially at  $x=0$ , and at  $t=0$  it is given a sharp kick to give it an initial speed  $v_0$  along the wire in the  $+x$  direction.

- Using an energy approach, derive an expression for the speed of the bead as a function of  $x$ .
- Determine the maximum value of  $x$ , in terms of the given parameters  $a$ ,  $b$ , and  $v_0$ . Also evaluate this expression for the special case  $v_0 = \sqrt{2ga}$