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PH2201 – Intermediate Mechanics I
Study Guide 3

Readings

Topic	Book/section	Comments
Momentum for system of particles	Kleppner 4.1-4.2	The form $\mathbf{F}_{ext} = d\mathbf{P}/dt$ of Newton's 2nd law is best for systems of particles.
Center of mass	Kleppner 4.3 Taylor 3.3 Morin 5.6	Study example 4.2 (skip pp. 121-124 on calculating the center of mass of extended objects)
Conservation of momentum and impulse	Kleppner 4.5-4.6 Taylor 3.1 Morin 5.5.1	Study examples 4.9, 4.11 *The principle of conservation of momentum is equivalent to Newton's 3rd law*
Systems with variable mass	Kleppner 4.7 Morin 5.8	study examples 4.12-4.14 *Although some processes are inherently inelastic (do not conserve kinetic energy), the total momentum of a closed system (no outside forces) is always conserved.*
Rocket propulsion	Kleppner 4.8 Taylor 3.2 Morin 5.5.2	Study examples 4.15-4.17. Note that the **u** in these examples is a vector $\mathbf{u} = -u\hat{x}$ for a rocket moving in the +x direction. In lecture I'll use the speed u in the rocket equations. *Rocket motion is an important example of systems with variable mass*

Practice Problems

Solutions provided in Morin -- **not to be turned in for grading**.

Morin Chapter 5:   problems 15, 16, 17, 21, 26, 27, 28

1. A rocket in free space that starts at rest with total mass M ejects exhaust at a given speed u. What is the mass of the rocket (including unused fuel) when its momentum is maximum? What is the mass when its energy is maximum? [use $E = \frac{1}{2} mv^2$ for energy]

2. You are riding on a sled that is given an initial push and slides across frictionless ice. Snow is falling vertically (in the frame of the ice) and hits the sled at a rate k (kg/s). The sled travels in tracks that constrain it to move in a straight line. At $t = 0$, the mass of the sled (including you) is M, and its speed is v_0. For the three situations below, determine the sled's speed v as a function of time. (Hint: see Morin pp. 158-159 for a qualitative discussion of this problem)

   a) You sweep the snow off the sled so that it leaves the sled in the direction perpendicular to the sled's tracks, as seen by you in the frame of the sled. No snow piles up on the sled.

   b) You sweep the snow off the sled so that it leaves the sled in the direction perpendicular to the sled's tracks, as seen by someone in the frame of the ice. No snow piles up on the sled.

   c) You do nothing (so the snow piles up on the sled)

3. A chain with length L and mass density σ kg/m is held in a heap, and you grab an end that protrudes a tiny bit out of the top. The chain is then released. As a function of time, what is the force that your hand must apply to keep the top end of the chain motionless? Assume that the chain has no friction with itself, so the remaining part of the heap is always in freefall. The situation at some random time t is shown in the figure. [hint: numerical factor in answer is not $\frac{1}{2}$]

4. A rocket (initial total mass m_0) needs to use its engines to hover stationary, just above the ground. (a)If it can afford to burn no more than a mass λm_0 of its fuel, for how long can it hover? [hint: Write down the condition that the thrust just balances the force of gravity. You can integrate the resulting equation by separating the variables t and m. Take the exhaust speed u to be constant.] (b)If $u = 3000$ m/s and $\lambda = 0.1$, for how long could the rocket hover just above the earth's surface?

5. The following comparison illustrates the advantage of multistage rockets. (a) A rocket of total mass m_0 carries 60% of its initial mass as fuel (i.e., the mass of fuel is 0.6 m_0). What is the rocket's final speed, accelerating from rest in free space, if it burns all its fuel in a single stage? Express your answer as a multiple of the exhaust speed u. (b) Suppose instead it burns the fuel in two stages as follows: In the first stage it burns a mass 0.3 m_0 of fuel. It then releases the first-stage fuel tank, which has a mass of 0.1 m_0, and finally burns the remaining 0.3 m_0 of fuel. Find the final speed in this case, assuming the same value of u throughout, and compare.

6. Kleppner 4.11   (use k instead of b for the rate of sand flow)

7. A single-stage rocket has been developed, which has a gas exhaust speed of 4000 m/s. The total mass of the rocket is 10^5 kg, with 90% of its mass being fuel. The fuel burns quickly in 100 s at a constant rate. For testing purposes, the rocket is launched vertically at rest from the Earth's surface. You can neglect air resistance and assume the acceleration of gravity is constant.
   a. After 100 s, how fast is the rocket moving?
   b. After 100 s, how high is the rocket?
   c. How high does the rocket eventually go?