Pendulum Oscillations

Overview

A pendulum behaves like a simple harmonic oscillator if its swing amplitude is small. For small amplitude, the restoring force F is very nearly proportional to the displacement θ. We say that F is linear in θ. Recall that a simple harmonic oscillator by definition has a restoring force that is proportional to displacement. However, for a pendulum, as amplitude is increased, the restoring force becomes increasingly non-linear in θ. To be precise, F is proportional to $\sin\theta$. As a result, the oscillation frequency varies with amplitude.

Another aspect of the pendulum is that it has two degrees of freedom. To see this we view the pendulum from above, and plot its position in the x, y plane of the table top. The pendulum can move linearly along any line in the x, y plane that passes through the origin (the equilibrium position). The pendulum is also capable of circular and elliptical motion. If the pendulum arm is segmented so that different swing frequencies exist along the x and y axes, the pendulum sweeps out Lissajous figures.

In this lab, you demonstrate a simple pendulum's non-linearity by measuring its frequency dependence on amplitude, and quantify this dependence in terms of ω_0, the small amplitude frequency. You also induce motions in a "double-jointed" pendulum to produce Lissajous figures in Logger Pro's x, y plot, and relate the parameters of the figures to the pendulum frequencies.

Setup and Procedure for Non-linear Oscillations

Set your force sensor range to 10N, mount the sensor on the stand, and hang the simple pendulum from the sensor as shown in the figure. Secure the string to the bob with a small piece of duct tape. Run Logger Pro, zero the force sensor, give the pendulum an initial small displacement $\theta_{max} \sim 10°$, measure the displacement, release the pendulum, and collect data for several seconds. The force plot should be periodic with frequency ω_{meas}. Fit the first three cycles of the force plot to a sine function. Read the frequency ω_{meas} from the fit parameters.

(W1) Record the initial displacement θ_{max} and the frequency ω_{meas} .

(W2) Is ω_{meas} the oscillation frequency of the pendulum? Explain.

(W3) Why use only the first three cycles for the fit?

Repeat for an initial displacement of approximately 80 degrees. Be careful during large swings! The pendulum processes, watch it and stop it to prevent it from striking the monitor or other.

(W4) Again record the initial displacement and frequency. For the two recorded sets of values, calculate the frequency $\omega_{theory}(\theta_{max}) = 2\pi/T_{theory}$ predicted by Y&F Eq. 13.35

$$T_{theory}(\theta_{max}) = 2\pi\sqrt{\frac{L}{g}}\left[1 + \frac{1}{4}sin^{\mathbf{2}}\left(\frac{\theta_{max}}{2}\right) + \frac{9}{64}sin^{\mathbf{4}}\left(\frac{\theta_{max}}{2}\right) + \cdots\right]$$

For the large amplitude, you need to include at least the first of the higher order terms of Eq. 13.35 to ensure $\omega_{theory}(\theta_{max})$ is computed accurately.

(W5) Estimate the amplitude at which the frequency differs from ω_0 by 5%.

Setup and Procedure for Lissajous Pendulum

Hang the Lissajous pendulum and carefully align two position sensors along the x and y axes, respectively, $20\ cm = 8\ in$ away from the pendulum box. Run Logger Pro, choose the X, Y plot, zero the sensors, induce small motions of the pendulum box (x, y amplitudes of about 1.0 cm), and start collection. Collect data for a minute or so, allowing Logger Pro to sketch the Lissajous figure. When you induce motion, be careful not to induce a twisting motion in the pendulum box. If you fail to produce a decent plot on the first attempt, keep trying. Together the Lissajous pendulum and Logger Pro are fussy, so you must have a soft touch and be persistent.

(W6) Without using the Lissajous figure, can you determine the two frequencies of the pendulum? You might consider measuring the pendulum's two lengths, or measuring the frequencies of linear motions along x and y separately using curve fits to sinusoids.

(W7) Can you determine the frequencies from the Lissajous plot and compare these to the values from W6? You might consider running the Lissajous application demonstrated in lecture, and try various frequencies to see if you can reproduce your Lissajous figure. You might also try to find a way to perform a curve fit to the Lissajous plot.