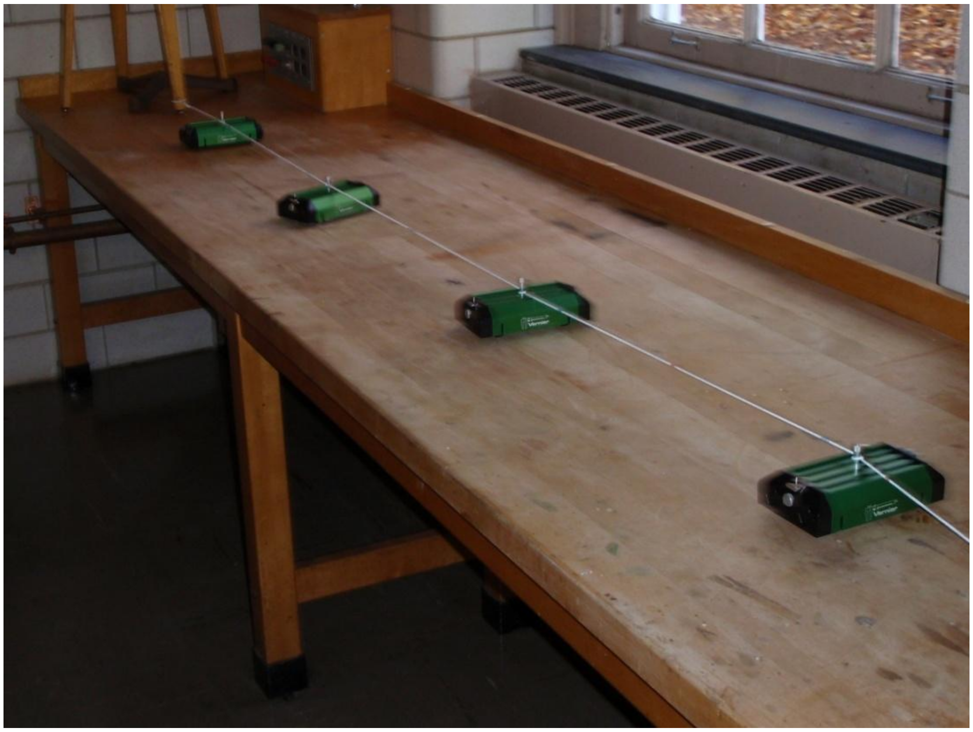
The Coupled Oscillations of Carts on an Elastic String



Overview

The motions of coupled oscillators differ from superimposed oscillations in so much as the coupled oscillators are dependent, that is they affect one another's motion. As a result, their combined motions are not a simple sum (superposition) of independent motions. We are surprised then to discover that a coupled system has what are called "normal modes" of oscillation, which are distinct and independent from one another, yet can be superimposed to create more complex motions. This said, perhaps the most fascinating thing we learn from the study of coupled oscillators is that wave motion (which we study in the second half of this course) can be thought of as arising from the motions of a system consisting of a many coupled oscillators. But even for the simplest of coupled systems, for example four carts, we begin to see hints of wave motion.

In this lab we assemble and study a system of coupled oscillators by attaching carts to an elastic string and setting the carts in motion. We create the normal modes of oscillation of the system by judicious choice of initial conditions, and then create motions that are each a superposition of normal modes. Finally, impressed with our accomplishments we become capricious, and through a somewhat arbitrary and impulsive shove of a single cart we suddenly see something new, a motion resembling a wave.

Setup

Be gentle with the carts, especially the wheels. Throughout the experiment, keep the amplitudes small so that the carts do not roll off the table!

Four Cart Setup

Attach four carts to the un-stretched string at positions $(1/8)L$, $(3/8)L$, $(5/8)L$ and $(7/8)L$, where L is the un-stretched length of the elastic string. Stretch the string and attach the ends to bench posts. Align the cart wheels parallel to the string so that the carts can find their equilibrium positions. Then align the carts transverse to the string, as shown in the figure.

Informal Procedure

Spend some time playing with the system to develop a feel for the motions of coupled oscillators. Apply various initial transverse displacements and release the carts. Apply various initial transverse shoves. Move a cart back and forth continuously and transversely and observe the effect on the other carts. Do this at various frequencies.

Formal Procedure

Apply initial displacements to the carts to create all possible transverse normal modes.

(W1) How many normal modes are possible with four carts?

(W2) Sketch each mode and indicate with cart positions the required initial displacements.

Apply initial velocities to the carts to create the normal modes.

(W3) Sketch each mode and indicate with arrows the required initial velocities.

Oscillating only one cart with your hand, create each of the normal modes.

(W4) What single parameter of the applied motion enables you to do this?

(W5) Measure and record the value of this parameter for each of the modes.

(W6) How does the system "know" how to find its normal modes?

Move one cart transversely to and fro *at very low frequency* and observe the system's motion. Move one cart transversely to and fro *at very high frequency* ***(gently!)*** and observe the system's motion. For both cases:

(W7) Sketch and describe the system's motion.

(W8) Explain why the system moves in this way.

Try to create one or more of the *longitudinal* normal modes. Keep the amplitudes *small!*

(W9) Sketch each mode and indicate with arrows the motions of the carts.

(W10) Compare the longitudinal and transverse modes and comment on their differences and similarities.