

PH1140 D09, Homework 5 - Waves on Surfaces and Membranes [310 points]

Due April 17, 2009 in lecture, and please put your homework in the 1140D boxes outside OH 107 (note the section numbers on the boxes).

1. [130] Membrane Wave. The wave function given by $\zeta(x, z, t) = A \cos(k_x x + k_z z - \omega t + \phi)$ describes the displacement of a plane wave moving along a membrane of mass density $\mu = 0.181 \text{ kg/m}^2$, where $A = 2.7 \text{ cm}$, $\omega = 22\pi \text{ rad/s}$, $k_x = 30 \text{ rad/m}$, $k_z = 40 \text{ rad/m}$, $\phi = \pi/12$.

- a) [20] Write the wave vector in rectangular and polar form. What is the direction of the wave?
- b) [30] What are the wave length, wave number and wave speed?
- c) [20] A wave front is a line of constant phase defined by $\theta = k_x x + k_z z - \omega t + \phi = \text{constant}$. At the time $t = (1/33) \text{ s}$, graph in the $\{x, z\}$ plane two consecutive wave fronts defined respectively by the two constant phases $\theta = 0$ and $\theta = 2\pi$.
- d) [10] Indicate on the graph the direction of the waves and the wave length.
- e) [20] Calculate the distance from the $\{x, z\}$ origin to the nearest of the two wave fronts, and indicate this distance on the graph.
- f) [20] Find the maximum transverse speed and acceleration of a point on the membrane.
- g) [10] What is the tension per unit length in the membrane?

2. [80] Ocean Waves. Tabulate the wave speed $v(k, h)$, frequency $\omega(k, h)$ and displacement $\eta(k, h)$ of ocean waves for the wave lengths and ocean depths given below. Suggestion: you can work this problem quickly by entering the formulas for these quantities into a Microsoft Excel spreadsheet, where you can utilize the built-in hyperbolic trigonometric functions $\sinh(kh)$ and $\tanh(kh)$. In the formula for displacement, use $A = 1 \text{ m}^2/\text{s}$.

$\lambda \text{ (m)}$	$h \text{ (m)}$
1	0.1, 1.0, 10, 100
1000	10, 100, 1000, 10000

3. [80] Ocean Waves. A seismic event occurs beneath the ocean at a place where the ocean depth is $h_{\text{source}} = 3000 \text{ m}$. The event creates a train of traveling ocean plane waves of amplitude $\eta_{\text{max}}(h_{\text{source}}) = 1.0 \text{ m}$, at a wave length of $\lambda_{\text{source}} = 6000 \text{ m}$. When wave train reaches a small boat located where the depth is $h_{\text{boat}} = 1000 \text{ m}$, the wave length has shortened to $\lambda_{\text{boat}} = 5070 \text{ m}$.

- a) [10] The waves arrive at the boat with frequency f_{boat} . How is this frequency related to the frequency f_{source} of the waves at the source?
- b) [30] Determine the frequency f_s , wave number k_s and wave speed v_s at the source.
- c) [30] Determine the frequency f_b , wave number k_b and wave speed v_b at the boat.
- d) [10] Determine the amplitude $\eta_{\text{max}}(h_{\text{boat}})$ of the waves at the boat.

4. [20] Show that the two traveling waves $\zeta(x, z, t) = A \cos(k_x x + k_z z - \omega t + \phi)$ and $\zeta(x, z, t) = A \cos(k_x x + k_z z + \omega t + \phi)$ combine to produce a standing wave. Hint: apply trigonometric identities involving the sum of two angles, and treat each of the wave phases as a sum of two angles.

5. [80] Standing Waves and Modes on Rectangular and Circular Membranes.

Use the two web applets <http://www.falstad.com/circosc/> and <http://www.falstad.com/membrane/> to study the modes of rectangular and circular membranes. Read the Full Directions to learn how to adjust magnitude and phase.

For the rectangular membrane, you use rectangular coordinates x and y to indicate the positions of nodes and anti-nodes. Also, use O's and X's to distinguish between crests and troughs.

For the circular membrane, you use polar coordinates r and φ to indicate the positions of nodes and anti-nodes. Use circles and dotted circles to distinguish between radial crests and troughs. Use O's and X's to distinguish between angular crests and troughs.

- a) [20]** For $n_x = 1$ and $n_y = 2$, sketch the mode of the rectangular membrane.
- b) [20]** For $n_x = 2$ and $n_y = 2$, sketch the mode of the rectangular membrane.
- c) [20]** For $n_r = n = 2$ and $n_\varphi = m = 0$, sketch the mode of the circular membrane.
- d) [20]** For $n_r = n = 1$ and $n_\varphi = m = 1$, sketch the mode of the circular membrane.