PH1140 D09 **Homework 4 [390 points]** **Due Thursday April 9, 2009**

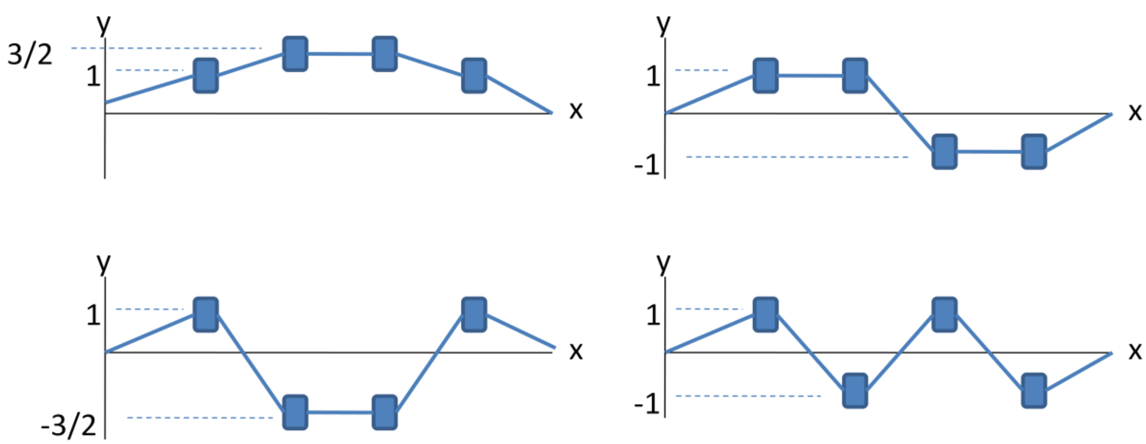
The numbers in brackets [ ] are the grade point values.

1. [50] Working with the A_{pn} and $y_{pn}(t)$ from Lecture (also see Study Guide 2). For a coupled system of $N = 3$ carts:

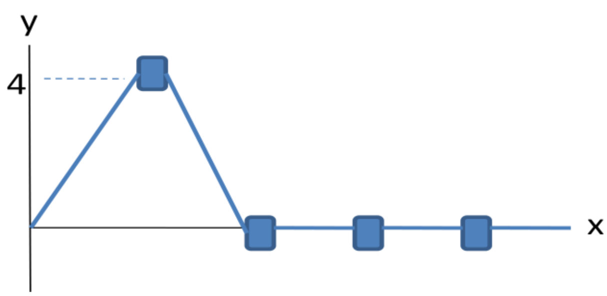
a) [30] Determine the transverse amplitudes A_{pn} and the displacements $y_{pn}(t)$ for carts $p = 1,2,3$, in each of the modes $n = 1,2,3$. Also determine the frequencies ω_n, wave numbers k_n, wave lengths λ_n and wave speeds v_n for each of the modes.
b) [10] Sketch the modes with carts fully displaced.
c) [10] Sketch the modes with carts in their equilibrium positions, and indicate the cart velocities with arrows.

2. [30] Transverse Oscillations of Four Carts. From your experience in lab, you know that four carts on elastic string have four transverse normal oscillation modes. Suppose that the four modes can be created, respectively, by the four sets of initial displacements shown below. For example, reading the mode one diagram (first diagram), the cart displacements are

$$y_1 = 1, \qquad y_2 = \frac{3}{2}, \qquad y_3 = \frac{3}{2}, \qquad y_4 = 1.$$



a) [10] Add the displacements of all four modes together to show that the combination of the four sets of displacements is equivalent to the set of displacements shown below.



From part (a), it is clear that the initial displacement of a single cart is equivalent to a sum of initial displacements, and "contains" a little bit of each of the four normal modes. In lab, you observed how, when a single cart is given an initial displacement and released, its motion is carried to the second cart, then the third, and finally the fourth, forming wave motion. This wave motion is a superposition of the normal modes, and is therefore a superposition of traveling waves involving four different wave speeds.

b) [20] Starting with the four carts in their equilibrium positions, you displace the first cart and release it to initiate transverse motion. Assume uniform cart spacing l such that the string length is given by $L = (N+1)l$, where $N = 4$. In terms of the string tension T, the cart spacing l, and the cart mass m, how long does it take for the fourth cart to "feel" the motion?

3. [30] Y&F Exercise 15.1. Sound waves have many characteristics in common with waves on a string. A sound wave, like a wave on a string, has a period T, frequency f, wave length λ, wave number k and wave speed v. The following familiar formulas also apply to sound waves:

$$T = \frac{1}{f}, \qquad \omega = 2\pi f, \qquad k = \frac{2\pi}{\lambda}, \qquad v = \frac{\omega}{k} = \lambda/T.$$

c) [10] What are the wave numbers of the waves in parts (a) and (b)?

4. [30] Y&F Exercise 15.5. Light behaves like a wave, having the same characteristics given above, namely T, f, ω, k and v.

c) [10] What are the wave numbers of the waves in part (a)?

5. [40] Y&F Exercise 15.12.

d) [10] Also show that the forms of eq. 15.3 in part (a) are equivalent to

$$y(x,t) = A\cos(kx - \omega t).$$

6. [20] Y&F Exercise 15.14.

7. [70] Y&F Exercise 15.24.

8. [50] Y&F Exercise 15.28 (merging of triangular pulses, section 15.6).

9. [40] Y&F Exercise 15.34 (standing waves, section 15.7).

10. [30] Y&F Exercise 15.39 (normal modes of a string, section 15.8).

Suggested exercises: Other exercise from Y&F Chapter 15, sections 2 through 8