PH1140 D09 **Homework 3 [210 points]** **Due Thursday April 2, 2009**

The numbers in brackets [ ] are the grade point values.

Non-linear Oscillations

1. [30] Y&F 13.48.

Super-Imposed Oscillations

2. [20] You wish to construct a Lissajous pendulum that "visits" the four corners of a rectangle once a minute. The rectangle has length a and width b.

a) What amplitudes A_1 and A_2 are required?

b) What minimum frequencies f_1 and f_2 are required?

3. [60] You hang two pendulums of different lengths L_1 and L_2 in close proximity and from different heights so that the pendulum bobs "touch" once a minute. In one minute, the pendulums complete n_1 and n_2 cycles, respectively.

a) [30] Determine the lengths in terms of n_1 and n_2.

b) Give three idealizations that are required for "touches" to occur.

c) Give a set of initial conditions that ensure touches occur.

d) Why must the ratio n_1/n_2 be a rational fraction?

Coupled Oscillations

4. [50] Two identical pendulums A and B are connected by a spring of force constant $k = 1.017\ N/m$. Each pendulum has a length of $L = 0.4\ m$ and a mass of $m = 0.23\ kg$. Neglect the mass of the spring, and use gravitational acceleration $g = 9.8 m/s^2$.

a) What are the periods of the two normal oscillation modes of the coupled pendulums?

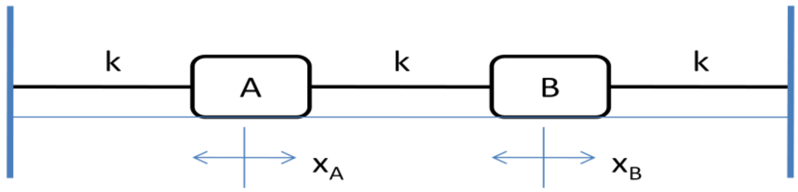
b) For the initial conditions $x_A = 0.02m,\ x_B = 0.02m,\ v_A = 0,\ v_B = 0$, determine the amplitudes and initial phases of the pendulum displacements $x_A(t)$ and $x_B(t)$. Hint: only one normal mode is involved.

c) For the initial conditions $x_A = 0,\ x_B = 0,\ v_A = 0.173 m/s,\ v_B = -0.173\ m/s$, determine the amplitudes and initial phases of the pendulum displacements $x_A(t)$ and $x_B(t)$. Hint: only one normal mode is involved.

d) Starting with both pendulums in their equilibrium positions, one pendulum is given an initial displacement and then released. What is the time interval T_{maxima} between successive maximum amplitudes of pendulum A? Hint: try using the coupled pendulum applet to study the motion, and consider the beat period as arising from the superposition of the two normal modes.

e) Sketch $x_A(t)$ and $x_B(t)$ over the time interval T_{maxima} .

5. [50] Longitudinal Oscillations of Two Carts. Two carts A and B, both of mass m, are attached to three identical springs of force constant k, all mounted between two fixed posts. The carts are shown in their equilibrium positions in the figure below. The carts may oscillate longitudinally, that is, horizontally left and right. Let their displacements be x_A and x_B.



When displaced, the carts experience forces from the springs.

a) [30] Using free body diagrams and Newton's second law, sum the spring forces acting on each cart and show that

$$m\ddot{x}_A = -kx_A + k(x_B - x_A)$$

$$m\ddot{x}_B = -kx_B - k(x_B - x_A).$$

The carts are coupled oscillators, and the above equations of motion are coupled, that is, both involve x_A and x_B. There are two normal oscillation modes. In the first mode the carts have the same motion and satisfy the condition $x_A = x_B$. In the second mode the carts have opposite motions and satisfy the condition $x_A = -x_B$.

b) Substitute the first condition into the two equations of motion to decouple them, that is, form two equations, one involving only x_A and the other involving only x_B. From these, obtain the oscillation frequency ω_1 of the first mode.

c) Substitute the second condition into the two equations of motion to decouple them and obtain the oscillation frequency ω_2 of the second mode.

Suggested <u>exercises</u>: French, exercises 2-3 through 2-6, and 5-1 through 5-6.