PH1140 D09, Homework 7 Solution [250 points]

1. [30] Y&F 32.10. Fields.

32.10. **IDENTIFY:** Apply Eqs.(32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

SET UP: The wave in this problem has a different phase, so $B_y(z,\ t) = B_{max}\sin(kx + \omega t)$.

EXECUTE: **(a)** The phase of the wave is given by $kx + \omega t$, so the wave is traveling in the $-x$ direction.

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c}$. $f = \dfrac{kc}{2\pi} = \dfrac{(1.38\times10^4\ \text{rad/m})(3.0\times10^8\ \text{m/s})}{2\pi} = 6.59\times10^{11}\ \text{Hz}.$

(c) Since the magnetic field is in the $+y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the $+z$-direction so that $\vec{E}\times\vec{B}$ will be in the $-x$-direction.

$\vec{E}(x,\ t) = +cB(x,\ t)\hat{k} = cB_{max}\sin(kx+\omega t)\hat{k}.$

$\vec{E}(x,\ t) = (c(3.25\times10^{-9}\ \text{T}))\sin\left((1.38\times10^4\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t\right)\hat{k}.$

$\vec{E}(x,\ t) = +(2.48\ \text{V/m})\sin\left((1.38\times10^4\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t\right)\hat{k}.$

EVALUATE: $\vec{E}$ and $\vec{B}$ have the same phase and are in perpendicular directions.

2. [10] Y&F 32.18. Power.

32.18. **IDENTIFY:** The intensity of the electromagnetic wave is given by Eq.(32.29): $I = \frac{1}{2}\epsilon_0 cE_{max}^2 = \epsilon_0 cE_{rms}^2$. The total energy passing through a window of area A during a time t is IAt.

SET UP: $\epsilon_0 = 8.85\times10^{-12}$ F/m

EXECUTE: $\text{Energy} = \epsilon_0 cE_{rms}^2 At = (8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})(0.0200\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 15.9\ \mu\text{J}$

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

3. [30] Y&F 32.35. Standing Wave.

32.35. **IDENTIFY:** The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $\left(2\frac{1}{2}\right)\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: **(a)** Since $\left(2\frac{1}{2}\right)\lambda = L$, we have $L = (5/2)(12.2\ \text{cm}) = 30.5$ cm.

(b) Solving for the frequency gives $f = c/\lambda = (3.00\times10^8\ \text{m/s})/(0.122\ \text{m}) = 2.46\times10^9$ Hz.

(c) $L = 35.5$ cm in this case. $\left(2\frac{1}{2}\right)\lambda = L$, so $\lambda = 2L/5 = 2(35.5\ \text{cm})/5 = 14.2$ cm.

$$f = c/\lambda = (3.00\times10^8\ \text{m/s})/(0.142\ \text{m}) = 2.11\times10^9\ \text{Hz}$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

4. [20] Y&F 33.24. Dispersion.

33.24. **IDENTIFY:** Snell's law is $n_a\sin\theta_a = n_b\sin\theta_b$. $v = \dfrac{c}{n}$.

SET UP: $a = \text{air}$, $b = \text{glass}$.

EXECUTE: **(a)** red: $n_b = \dfrac{n_a\sin\theta_a}{\sin\theta_b} = \dfrac{(1.00)\sin57.0^\circ}{\sin38.1^\circ} = 1.36$. violet: $n_b = \dfrac{(1.00)\sin57.0^\circ}{\sin36.7^\circ} = 1.40$.

(b) red: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8\ \text{m/s}}{1.36} = 2.21\times10^8\ \text{m/s}$; violet: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8\ \text{m/s}}{1.40} = 2.14\times10^8\ \text{m/s}$.

EVALUATE: n is larger for the violet light and therefore this light is bent more toward the normal, and the violet light has a smaller speed in the glass than the red light.

5. [20] Y&F 33.28. Polarization.

33.28. **IDENTIFY:** Set $I = I_0/10$, where I is the intensity of light passed by the second polarizer.

SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

EXECUTE: **(a)** After the first filter $I = \frac{I_0}{2}$ and the light is polarized along the vertical direction. After the second filter we want $I = \frac{I_0}{10}$, so $\frac{I_0}{10} = \left(\frac{I_0}{2}\right)(\cos\phi)^2$. $\cos\phi = \sqrt{2/10}$ and $\phi = 63.4°$.

(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter $\frac{I_0}{10} = I_0(\cos\phi)^2$. $\cos\phi = \sqrt{1/10}$ and $\phi = 71.6°$.

EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.

6. [20] Y&F 35.33. Thin Film.

35.33. **IDENTIFY:** Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.

(a) SET UP: Consider Figure 35.33.

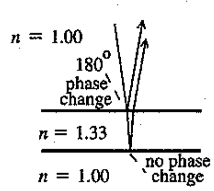

Figure 35.33

There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

EXECUTE: The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to the reflections. Hence the condition for constructive interference is $2t = \left(m + \frac{1}{2}\right)(\lambda_0/n)$, $m = 0,1,2,\ldots$ Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(290\text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{771.4\text{ nm}}{m+\frac{1}{2}}$$

for $m = 0$, $\lambda = 1543$ nm; for $m = 1$, $\lambda = 514$ nm; for $m = 2$, $\lambda = 308$ nm;... Only 514 nm is in the visible region; the color for this wavelength is green.

(b) $$\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(340\text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{904.4\text{ nm}}{m+\frac{1}{2}}$$

for $m = 0$, $\lambda = 1809$ nm; for $m = 1$, $\lambda = 603$ nm; for $m = 2$, $\lambda = 362$ nm;... Only 603 nm is in the visible region; the color for this wavelength is orange.

EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

7. [20] Y&F 36.32. Grating.

36.32. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.

SET UP: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^5\ \text{m}^{-1}} = 2.86\times10^{-6}$ m

EXECUTE: $m=1$: $\theta_{400} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{4.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 8.05°$.

$\theta_{700} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{7.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 14.18°$. $\Delta\theta_1 = 14.18° - 8.05° = 6.13°$.

$m=3$: $\theta_{400} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(4.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 24.8°$.

$\theta_{700} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(7.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 47.3°$. $\Delta\theta_1 = 47.3° - 24.8° = 22.5°$.

EVALUATE: $\Delta\theta$ is larger in third order.

8. [30] Y&F 38.2. Photons.

38.2. **IDENTIFY** and **SET UP:** $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00\times10^8$ m/s. 1 eV $= 1.60\times10^{-16}$ J.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{505\times10^{-9}\ \text{m}} = 5.94\times10^{14}$ Hz

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(5.94\times10^{14}\ \text{Hz}) = 3.94\times10^{-19}\ \text{J} = 2.46$ eV

(c) $K = \tfrac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(3.94\times10^{-19}\ \text{J})}{9.5\times10^{-15}\ \text{kg}}} = 9.1$ mm/s

9. [30] Y&F 39.9. Photon versus Electron.

39.9. **IDENTIFY** and **SET UP:** A photon has zero mass and its energy and wavelength are related by Eq.(38.2). An electron has mass. Its energy is related to its momentum by $E = p^2/2m$ and its wavelength is related to its momentum by Eq.(39.1).

EXECUTE: **(a)** <u>photon</u>

$E = \dfrac{hc}{\lambda}$ so $\lambda = \dfrac{hc}{E} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{(20.0\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})} = 62.0$ nm

<u>electron</u>

$E = p^2/(2m)$ so $p = \sqrt{2mE} = \sqrt{2(9.109\times10^{-31}\ \text{kg})(20.0\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})} = 2.416\times10^{-24}$ kg·m/s

$\lambda = h/p = 0.274$ nm

(b) <u>photon</u> $E = hc/\lambda = 7.946\times10^{-19}\ \text{J} = 4.96$ eV

<u>electron</u> $\lambda = h/p$ so $p = h/\lambda = 2.650\times10^{-27}$ kg·m/s

$E = p^2/(2m) = 3.856\times10^{-24}\ \text{J} = 2.41\times10^{-5}$ eV

(c) **EVALUATE:** You should use a probe of wavelength approximately 250 nm. An electron with $\lambda = 250$ nm has much less energy than a photon with $\lambda = 250$ nm, so is less likely to damage the molecule. Note that $\lambda = h/p$ applies to all particles, those with mass and those with zero mass. $E = hf = hc/\lambda$ applies only to photons and $E = p^2/2m$ applies only to particles with mass.

10. [40] Y&F 39.13. Electron Diffraction.

39.13. **(a)** $\lambda = 0.10\,\text{nm}$. $p = mv = h/\lambda$ so $v = h/(m\lambda) = 7.3\times10^6$ m/s.

(b) $E = \frac{1}{2}mv^2 = 150\,\text{eV}$

(c) $\text{E} = hc/\lambda = 12$ KeV

(d) The electron is a better probe because for the same λ it has less energy and is less damaging to the structure being probed.