PH1140 D09, Homework 6 [260 points]

Due April 24, 2009 in lecture. Please put your homework in the 1140D boxes outside OH 107 (note the section numbers on the boxes).

1. [30] A train of 2D line waves of wavelength $\lambda = 0.59\ cm$ travel in the $+x$ direction, strike barrier at $x = 0$ and pass through two small slits located at $y = \pm 8.5\ cm$. Consider the interference pattern created by the waves at distances $r \gg d$.

a) [10] Determine the 7th angle at which the waves from the two slits combine constructively.
The first angle is $\theta = 0$, corresponding to $n = 0$. The 7th angle corresponds to $n = 6$:

$$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right) = 0.2098\, rad = 12.02°\,.$$

b) [10] Determine the 7th angle at which the waves from the two slits combine destructively.
The 7th angle corresponds to $n = 6$:

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\left[n + \frac{1}{2}\right]\right) = 0.2256\, rad = 12.92°\,.$$

c) [10] What is the path difference r_{dif} and phase difference φ_{dif} at the location $x = 3.8\ m, y = 0.7\ m$?

The path difference is $r_{dif} = |r_2 - r_1| = \left|\sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - x)^2}\right|$ where $\{x_1, y_1\}$ and $\{x_2, y_2\}$ are the coordinates of the two slits, thus

$$r_{dif} = 0.03079\, m = 5.219\, \lambda\,.$$

The phase difference is then

$$\varphi_{dif} = 2\pi \frac{r_{dif}}{\lambda} = 32.79\, rad = 1.38\, rad\,.$$

2. [30] An inbound traveling circular wave on a membrane has amplitude $A = 0.2\ cm$ and wavelength $\lambda = 1.49\ cm$ at $r = 19\ cm$.

a) [10] Calculate the average power density of the wave if the membrane tension is $\mathcal{F} = 3.5\ N/m$ and the membrane mass density is $\sigma = 26\ g/m^2$.
Since $r \gg \lambda$ we may use the approximate formula

$$P_{avg} = \sqrt{\sigma\mathcal{F}}\frac{\omega^2 A^2}{2r}\,, \qquad where\ \ \omega = kv = \frac{2\pi}{\lambda}\sqrt{\frac{\mathcal{F}}{\sigma}} = 4893\, rad/s\,.$$

Then

$$P_{avg}\,(r = 19cm) = 76.0\, W/m\,.$$

b) [10] What is the average power density at $r = 27\ cm$?
Since the power density varies inversely with r,

$$P_{avg}\,(r = 27cm) = P_{avg}\,(r = 19cm)\frac{19}{27} = 53.5\, W/m\,.$$

c) [10] What is the average power density at $r = 14\ cm$?

$$P_{avg}\,(r = 14cm) = P_{avg}\,(r = 19cm)\frac{19}{14} = 103\, W/m\,.$$

3D Sound Waves

3. [30] Work Y&F Exercise 16.1.

16.1. **IDENTIFY** and **SET UP:** Eq.(15.1) gives the wavelength in terms of the frequency. Use Eq.(16.5) to relate the pressure and displacement amplitudes.

EXECUTE: **(a)** $\lambda = v/f = (344 \text{ m/s})/1000 \text{ Hz} = 0.344 \text{ m}$

(b) $p_{max} = BkA$ and Bk is constant gives $p_{max1}/A_1 = p_{max2}/A_2$

$$A_2 = A_1\left(\frac{p_{max2}}{p_{max1}}\right) = 1.2\times10^{-8}\text{ m}\left(\frac{30\text{ Pa}}{3.0\times10^{-2}\text{ Pa}}\right) = 1.2\times10^{-5}\text{ m}$$

(c) $p_{max} = BkA = 2\pi BA/\lambda$

$$p_{max}\lambda = 2\pi BA = \text{constant so } p_{max1}\lambda_1 = p_{max2}\lambda_2 \text{ and } \lambda_2 = \lambda_1\left(\frac{p_{max1}}{p_{max2}}\right) = (0.344\text{ m})\left(\frac{3.0\times10^{-2}\text{ Pa}}{1.5\times10^{-3}\text{ Pa}}\right) = 6.9\text{ m}$$

$f = v/\lambda = (344 \text{ m/s})/6.9 \text{ m} = 50 \text{ Hz}$

EVALUATE: The pressure amplitude and displacement amplitude are directly proportional. For the same displacement amplitude, the pressure amplitude decreases when the frequency decreases and the wavelength increases.

4. [20] Work Y&F Exercise 16.5.

16.5. **IDENTIFY:** $v = f\lambda$. Apply Eq.(16.7) for the waves in the liquid and Eq.(16.8) for the waves in the metal bar.

SET UP: In part (b) the wave speed is $v = \dfrac{d}{t} = \dfrac{1.50\text{ m}}{3.90\times10^{-4}\text{ s}}$

EXECUTE: **(a)** Using Eq.(16.7), $B = v^2\rho = (\lambda f)^2\rho$, so $B = [(8\text{ m})(400\text{ Hz})]^2(1300\text{ kg/m}^3) = 1.33\times10^{10}\text{ Pa}$.

(b) Using Eq.(16.8), $Y = v^2\rho = (L/t)^2\rho = \left[(1.50\text{ m})/(3.90\times10^{-4}\text{ s})\right]^2(6400\text{ kg/m}^3) = 9.47\times10^{10}\text{ Pa}$.

EVALUATE: In the liquid, $v = 3200$ m/s and in the metal, $v = 3850$ m/s. Both these speeds are much greater than the speed of sound in air.

5. [30] Work Y&F Exercise 16.14.

16.14. **IDENTIFY:** The intensity I is given in terms of the displacement amplitude by Eq.(16.12) and in terms of the pressure amplitude by Eq.(16.14). $\omega = 2\pi f$. Intensity is energy per second per unit area.

SET UP: For part (a), $I = 10^{-12}\text{ W/m}^2$. For part (b), $I = 3.2\times10^{-3}\text{ W/m}^2$.

EXECUTE: **(a)** $I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$.

$$A = \frac{1}{\omega}\sqrt{\frac{2I}{\sqrt{\rho B}}} = \frac{1}{2\pi(1000\text{ Hz})}\sqrt{\frac{2(1\times10^{-12}\text{ W/m}^2)}{\sqrt{(1.20\text{ kg/m}^3)(1.42\times10^5\text{ Pa})}}} = 1.1\times10^{-11}\text{ m}. \quad I = \frac{p_{max}^2}{2\sqrt{\rho B}}.$$

$$p_{max} = \sqrt{2I\sqrt{\rho B}} = \sqrt{2(1\times10^{-12}\text{ W/m}^2)\sqrt{(1.20\text{ kg/m}^3)(1.42\times10^5\text{ Pa})}} = 2.9\times10^{-5}\text{ Pa} = 2.8\times10^{-10}\text{ atm}$$

(b) A is proportional to $\sqrt{I}$, so $A = (1.1\times10^{-11}\text{ m})\sqrt{\dfrac{3.2\times10^{-3}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}} = 6.2\times10^{-7}\text{ m}$. p_{max} is also proportional to $\sqrt{I}$, so $p_{max} = (2.9\times10^{-5}\text{ Pa})\sqrt{\dfrac{3.2\times10^{-3}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}} = 1.6\text{ Pa} = 1.6\times10^{-5}\text{ atm}$.

(c) area $= (5.00 \text{ mm})^2 = 2.5\times10^{-5} \text{ m}^2$. Part (a): $(1\times10^{-12} \text{ W/m}^2)(2.5\times10^{-5} \text{ m}^2) = 2.5\times10^{-17}$ J/s.
Part (b): $(3.2\times10^{-3} \text{ W/m}^2)(2.5\times10^{-5} \text{ m}^2) = 8.0\times10^{-8}$ J/s.

EVALUATE: For faint sounds the displacement and pressure variation amplitudes are very small. Intensities for audible sounds vary over a very wide range.

6. [20] Work Y&F Exercise 16.25.

16.25. IDENTIFY and SET UP: An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.

EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.25a. The open ends are displacement antinodes.

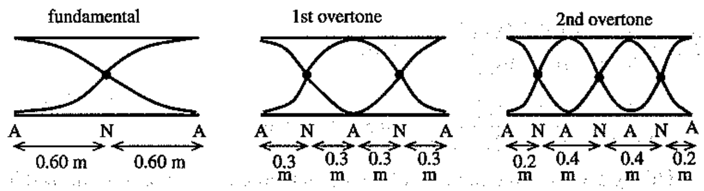

Figure 16.25a

Location of the displacement nodes (N) measured from the left end:
fundamental 0.60 m
1st overtone 0.30 m, 0.90 m
2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:
fundamental 0, 1.20 m
1st overtone 0, 0.60 m, 1.20 m
2nd overtone 0, 0.40 m, 0.80 m, 1.20 m
(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.25b.

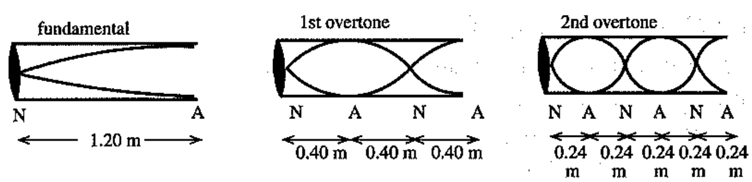

Figure 16.25b

Location of the displacement nodes (N) measured from the closed end:
fundamental 0
1st overtone 0, 0.80 m
2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:
fundamental 1.20 m
1st overtone 0.40 m, 1.20 m
2nd overtone 0.24 m, 0.72 m, 1.20 m
EVALUATE: The node-to-node or antinode-to-antinode distance is $\lambda/2$. For the higher overtones the frequency is higher and the wavelength is smaller.

7. [20] Work Y&F Exercise 16.31.

16.31. **IDENTIFY** and **SET UP:** Use the standing wave pattern to relate the wavelength of the standing wave to the length of the air column and then use Eq.(15.1) to calculate f. There is a displacement antinode at the top (open) end of the air column and a node at the bottom (closed) end, as shown in Figure 16.31

EXECUTE: **(a)**

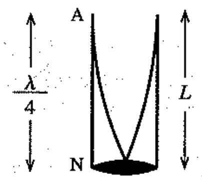

Figure 16.31

$$\lambda/4 = L$$
$$\lambda = 4L = 4(0.140\text{ m}) = 0.560\text{ m}$$
$$f = \frac{v}{\lambda} = \frac{344\text{ m/s}}{0.560\text{ m}} = 614\text{ Hz}$$

(b) Now the length L of the air column becomes $\frac{1}{2}(0.140\text{ m}) = 0.070\text{ m}$ and $\lambda = 4L = 0.280\text{ m}$.

$$f = \frac{v}{\lambda} = \frac{344\text{ m/s}}{0.280\text{ m}} = 1230\text{ Hz}$$

EVALUATE: Smaller L means smaller λ which in turn corresponds to larger f.

8. [30] Work Y&F Exercise 16.34.

16.34. **IDENTIFY:** Constructive interference occurs when the difference of the distances of each source from point P is an integer number of wavelengths. The interference is destructive when this difference of path lengths is a half integer number of wavelengths.

SET UP: The wavelength is $\lambda = v/f = (344\text{ m/s})/(206\text{ Hz}) = 1.67\text{ m}$. Since P is between the speakers, x must be in the range 0 to L, where $L = 2.00\text{ m}$ is the distance between the speakers.

EXECUTE: The difference in path length is $\Delta l = (L - x) - x = L - 2x$, or $x = (L - \Delta l)/2$. For destructive interference, $\Delta l = (n + (1/2))\lambda$, and for constructive interference, $\Delta l = n\lambda$.

(a) Destructive interference: $n = 0$ gives $\Delta l = 0.835\text{ m}$ and $x = 0.58\text{ m}$. $n = -1$ gives $\Delta l = -0.835\text{ m}$ and $x = 1.42\text{ m}$. No other values of n place P between the speakers.

(b) Constructive interference: $n = 0$ gives $\Delta l = 0$ and $x = 1.00\text{ m}$. $n = 1$ gives $\Delta l = 1.67\text{ m}$ and $x = 0.17\text{ m}$. $n = -1$ gives $\Delta l = -1.67\text{ m}$ and $x = 1.83\text{ m}$. No other values of n place P between the speakers.

(c) Treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls, ceiling, and floor.

EVALUATE: Points of constructive interference are a distance $\lambda/2$ apart, and the same is true for the points of destructive interference.

9. [20] Work Y&F Exercise 16.38.

16.38. **IDENTIFY:** $f_{\text{beat}} = |f_1 - f_2|$. $v = f\lambda$.

SET UP: $v = 344\text{ m/s}$, Let $\lambda_1 = 6.50\text{ cm}$ and $\lambda_2 = 6.52\text{ cm}$. $\lambda_2 > \lambda_1$ so $f_1 > f_2$.

EXECUTE: $f_1 - f_2 = v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} = \frac{(344\text{ m/s})(0.02\times10^{-2}\text{ m})}{(6.50\times10^{-2}\text{ m})(6.52\times10^{-2}\text{ m})} = 16\text{ Hz}$. There are 16 beats per second.

EVALUATE: We could have calculated f_1 and f_2 and subtracted, but doing it this way we would have to be careful to retain enough figures in intermediate calculations to avoid round-off errors.

10. [30] Work Y&F Exercise 16.43.

16.43. **IDENTIFY:** Apply the Doppler shift equation $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$.

SET UP: The positive direction is from listener to source. $f_S = 392$ Hz.

(a) $v_S = 0$. $v_L = -15.0$ m/s. $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s} - 15.0 \text{ m/s}}{344 \text{ m/s}}\right)(392 \text{ Hz}) = 375 \text{ Hz}$

(b) $v_S = +35.0$ m/s. $v_L = +15.0$ m/s. $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} + 35.0 \text{ m/s}}\right)(392 \text{ Hz}) = 371 \text{ Hz}$

(c) $f_{beat} = f_1 - f_2 = 4$ Hz

EVALUATE: The distance between whistle *A* and the listener is increasing, and for whistle *A* $f_L < f_S$. The distance between whistle *B* and the listener is also increasing, and for whistle *B* $f_L < f_S$.