PH1140 D09 Homework 2 Solution

1. [20] Y&F Exercise 13.25.

13.25. **IDENTIFY:** Use the results of Example 13.15 and also that $E = \frac{1}{2}kA^2$.

SET UP: In the example, $A_2 = A_1\sqrt{\frac{M}{M+m}}$ and now we want $A_2 = \frac{1}{2}A_1$. Therefore, $\frac{1}{2} = \sqrt{\frac{M}{M+m}}$, or $m = 3M$. For the energy, $E_2 = \frac{1}{2}kA_2^2$, but since $A_2 = \frac{1}{2}A_1$, $E_2 = \frac{1}{4}E_1$, and $\frac{3}{4}E_1$ is lost to heat.

EVALUATE: The putty and the moving block undergo a totally inelastic collision and the mechanical energy of the system decreases.

2. [20] For Y&F Exercise 13.56, let the oscillator be at maximum amplitude $A_0 = 1.44\ cm$ at time $t = 0$.

a) Find the energy $E(t)$ of the damped oscillator.

The initial energy is determined by the initial amplitude

$$E_0 = \frac{k}{2}A_0^2 = \frac{250\ N/m}{2}(1.44 \times 10^{-2}\ m)^2 = 0.0259\,J.$$

The energy $E(t)$ is given by (from lecture)

$$E(t) = E_0 e^{-\gamma t/2}$$

where

$$\gamma = \frac{b}{m} = \frac{13.3\ kg/s}{2.2\ kg} = 6.05\frac{rad}{s}.$$

b) At what point in time does E fall to half its initial value?

We have

$$E(t) = E_0 e^{-\gamma t/2} = \frac{E_0}{2} \qquad \rightarrow \qquad e^{-\gamma t/2} = \frac{1}{2}.$$

Taking the natural log and solving for t gives

$$t = -\frac{2}{\gamma}\ln\left(\frac{1}{2}\right) = 0.229\ s\,.$$

3. [20] Y&F Exercise 13.60.

13.60. **IDENTIFY:** Apply Eq.(13.46).

SET UP: $\omega_d = \sqrt{k/m}$ corresponds to resonance, and in this case Eq.(13.46) reduces to $A = F_{max}/b\omega_d$.

EXECUTE: **(a)** $A_1/3$

(b) $2A_1$

EVALUATE: Note that the resonance frequency is independent of the value of b. (See Figure 13.28 in the textbook).

4. [30] Y&F Exercise 13.61.

13.61 **IDENTIFY and SET UP:** Apply Eq.(13.46): $A = \dfrac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$

EXECUTE: **(a)** Consider the special case where $k - m\omega_d^2 = 0$, so $A = F_{max}/b\omega_d$ and $b = F_{max}/A\omega_d$. Units of $\dfrac{F_{max}}{A\omega_d}$ are

$\dfrac{\text{kg}\cdot\text{m/s}^2}{(\text{m})(\text{s}^{-1})} = \text{kg/s}$. For units consistency the units of b must be kg/s.

(b) Units of $\sqrt{km}$: $[(\text{N/m})\text{kg}]^{1/2} = (\text{N kg/m})^{1/2} = [(\text{kg}\cdot\text{m/s}^2)(\text{kg})/\text{m}]^{1/2} = (\text{kg}^2/\text{s}^2)^{1/2} = \text{kg/s}$, the same as the units for b.

(c) For $\omega_d = \sqrt{k/m}$ (at resonance) $A = (F_{max}/b)\sqrt{m/k}$.

(i) $b = 0.2\sqrt{km}$

$$A = F_{max}\sqrt{\frac{m}{k}}\frac{1}{0.2\sqrt{km}} = \frac{F_{max}}{0.2k} = 5.0\frac{F_{max}}{k}.$$

(ii) $b = 0.4\sqrt{km}$

$$A = F_{max}\sqrt{\frac{m}{k}}\frac{1}{0.4\sqrt{km}} = \frac{F_{max}}{0.4k} = 2.5\frac{F_{max}}{k}.$$

EVALUATE: Both these results agree with what is shown in Figure 13.28 in the textbook. As b increases the maximum amplitude decreases.

5. [30] Y&F Exercise 13.34.

13.34. **IDENTIFY:** The torsion constant κ is defined by $\tau_z = -\kappa\theta$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}}$ and $T = 1/f$. $\theta(t) = \Theta\cos(\omega t + \phi)$.

SET UP: For the disk, $I = \frac{1}{2}MR^2$. $\tau_z = -FR$. At $t = 0$, $\theta = \Theta = 3.34° = 0.0583$ rad, so $\phi = 0$.

EXECUTE: **(a)** $\kappa = -\dfrac{\tau_z}{\theta} = -\dfrac{-FR}{0.0583\text{ rad}} = +\dfrac{(4.23\text{ N})(0.120\text{ m})}{0.0583\text{ rad}} = 8.71\text{ N}\cdot\text{m/rad}$

(b) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{\kappa}{I}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2\kappa}{MR^2}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2(8.71\text{ N}\cdot\text{m/rad})}{(6.50\text{ kg})(0.120\text{ m})^2}} = 2.17\text{ Hz}$. $T = 1/f = 0.461$ s.

(c) $\omega = 2\pi f = 13.6$ rad/s. $\theta(t) = (3.34°)\cos([13.6\text{ rad/s}]t)$.

EVALUATE: The frequency and period are independent of the initial angular displacement, so long as this displacement is small.

6. [20] Y&F Exercise 13.36.

13.36. **IDENTIFY:** Eq.(13.24) and $T = 1/f$ says $T = 2\pi\sqrt{\dfrac{I}{\kappa}}$.

SET UP: $I = \frac{1}{2}mR^2$.

EXECUTE: Solving Eq. (13.24) for κ in terms of the period,

$$\kappa = \left(\frac{2\pi}{T}\right)^2 I = \left(\frac{2\pi}{1.00\text{ s}}\right)^2 ((1/2)(2.00\times10^{-3}\text{ kg})(2.20\times10^{-2}\text{ m})^2) = 1.91\times10^{-5}\text{ N}\cdot\text{m/rad}.$$

EVALUATE: The longer the period, the smaller the torsion constant.

7. [30] Y&F Exercise 30.38.

13.38. **IDENTIFY:** $\theta(t)$ is given by $\theta(t)=\Theta\cos(\omega t+\phi)$. Evaluate the derivatives specified in the problem.

SET UP: $d(\cos\omega t)/dt=-\omega\sin\omega t$. $d(\sin\omega t)/dt=\omega\cos\omega t$. $\sin^2\theta+\cos^2\theta=1$

In this problem, $\varphi=0$.

EXECUTE: **(a)** $\frac{d\theta}{dt}=-\omega\,\Theta\sin(\omega\,t)$ and $\alpha=\frac{d^2\theta}{dt^2}=-\omega^2\,\Theta\cos(\omega\,t)$.

(b) When the angular displacement is Θ, $\Theta=\Theta\cos(\omega t)$. This occurs at $t=0$, so $\omega=0$. $\alpha=-\omega^2\Theta$. When the angular displacement is $\Theta/2$, $\frac{\Theta}{2}=\Theta\cos(\omega t)$, or $\frac{1}{2}=\cos(\omega t)$. $\frac{d\theta}{dt}=\frac{-\omega\,\Theta\sqrt{3}}{2}$ since $\sin(\omega t)=\frac{\sqrt{3}}{2}$. $\alpha=\frac{-\omega^2\Theta}{2}$, since $\cos(\omega t)=1/2$.

EVALUATE: $\cos(\omega t)=\frac{1}{2}$ when $\omega t=\pi/3$ rad $=60°$. At this t, $\cos(\omega t)$ is decreasing and θ is decreasing, as required. There are other, larger values of ωt for which $\theta=\Theta/2$, but θ is increasing.

8. [20] Y&F Exercise 30.41.

13.41. **IDENTIFY:** $T=2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4}=\frac{\pi}{2}\sqrt{\frac{L}{g}}=0.25$ s.

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on L and g.