13.4. **IDENTIFY:** The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.

SET UP: The maximum x is 10.0 cm. The time for one cycle is 16.0 s.

EXECUTE: **(a)** $T = 16.0$ s so $f = \frac{1}{T} = 0.0625$ Hz.

(b) $A = 10.0$ cm.

(c) $T = 16.0$ s

(d) $\omega = 2\pi f = 0.393$ rad/s

EVALUATE: After one cycle the motion repeats.

13.11. **IDENTIFY:** Use Eq.(13.19) to calculate A. The initial position and velocity of the block determine ϕ. $x(t)$ is given by Eq.(13.13).

SET UP: $\cos\theta$ is zero when $\theta = \pm\pi/2$ and $\sin(\pi/2) = 1$.

EXECUTE: **(a)** From Eq. (13.19), $A = \left|\frac{v_0}{\omega}\right| = \left|\frac{v_0}{\sqrt{k/m}}\right| = 0.98$ m.

(b) Since $x(0) = 0$, Eq.(13.14) requires $\phi = \pm\frac{\pi}{2}$. Since the block is initially moving to the left, $v_{0x} < 0$ and Eq.(13.7) requires that $\sin\phi > 0$, so $\phi = +\frac{\pi}{2}$.

(c) $\cos(\omega t + (\pi/2)) = -\sin\omega t$, so $x = (-0.98 \text{ m})\sin((12.2 \text{ rad/s})t)$.

EVALUATE: The $x(t)$ result in part (c) does give $x = 0$ at $t = 0$ and $x < 0$ for t slightly greater than zero.

13.15. **IDENTIFY:** Apply $T = 2\pi\sqrt{\frac{m}{k}}$. Use the information about the empty chair to calculate k.

SET UP: When $m = 42.5$ kg, $T = 1.30$ s.

EXECUTE: Empty chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(42.5 \text{ kg})}{(1.30 \text{ s})^2} = 993$ N/m

With person in chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $m = \frac{T^2 k}{4\pi^2} = \frac{(2.54 \text{ s})^2(993 \text{ N/m})}{4\pi^2} = 162$ kg and $m_{\text{person}} = 162 \text{ kg} - 42.5 \text{ kg} = 120$ kg.

EVALUATE: For the same spring, when the mass increases, the period increases.

13.19. **IDENTIFY:** Compare the specific $x(t)$ given in the problem to the general form of Eq.(13.13).

SET UP: $A = 7.40$ cm, $\omega = 4.16$ rad/s, and $\phi = -2.42$ rad.

EXECUTE: **(a)** $T = \frac{2\pi}{\omega} = \frac{2\pi}{4.16 \text{ rad/s}} = 1.51$ s.

(b) $\omega = \sqrt{\frac{k}{m}}$ so $k = m\omega^2 = (1.50 \text{ kg})(4.16 \text{ rad/s})^2 = 26.0$ N/m

(c) $v_{\max} = \omega A = (4.16 \text{ rad/s})(7.40 \text{ cm}) = 30.8$ cm/s

(d) $F_x = -kx$ so $F = kA = (26.0 \text{ N/m})(0.0740 \text{ m}) = 1.92$ N.

(e) $x(t)$ evaluated at $t = 1.00$ s gives $x = -0.0125$ m. $v_x = -\omega A\sin(\omega t + \phi) = 30.4$ cm/s.

$a_x = -kx/m = -\omega^2 x = +0.216 \text{ m/s}^2$.

EVALUATE: The maximum speed occurs when $x = 0$ and the maximum force is when $x = \pm A$.

13.56. **IDENTIFY:** If the system is critically damped or overdamped it doesn't oscillate. With no damping, $\omega=\sqrt{m/k}$.

With underdamping, the angular frequency has the smaller value $\omega'=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$.

SET UP: $m=2.20\text{ kg}$, $k=250.0\text{ N/m}$. $T'=\frac{2\pi}{\omega'}$ and $\omega'=\frac{2\pi}{T'}=\frac{2\pi}{0.615\text{ s}}=10.22\text{ rad/s}$.

EXECUTE: **(a)** $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{250.0\text{ N/m}}{2.20\text{ kg}}}=10.66\text{ rad/s}$. $\omega'<\omega$ so the system is damped. $\omega'=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$ gives

$b=2m\sqrt{\frac{k}{m}-\omega'^2}=2(2.20\text{ kg})\sqrt{\frac{250.0\text{ N/m}}{2.20\text{ kg}}-(10.22\text{ rad/s})^2}=13.3\text{ kg/s}$.

(b) Since the motion has a period the system oscillates and is underdamped.

EVALUATE: The critical value of the damping constant is $b=2\sqrt{km}=2\sqrt{(250.0\text{ N/m})(2.20\text{ kg})}=46.9\text{ kg/s}$. In this problem b is much less than its critical value.

13.59. **IDENTIFY:** $x(t)$ is given by Eq.(13.42). $v_x=dx/dt$ and $a_x=dv_x/dt$.

SET UP: $d(\cos\omega' t)/dt=-\omega'\sin\omega' t$. $d(\sin\omega' t)/dt=\omega'\cos\omega' t$. $d(e^{-\alpha t})/dt=-\alpha e^{-\alpha t}$.

EXECUTE: **(a)** With $\phi=0$, $x(0)=A$.

(b) $v_x=\frac{dx}{dt}=Ae^{-(b/2m)t}\left[-\frac{b}{2m}\cos\omega' t-\omega'\sin\omega' t\right]$, and at $t=0$, $v_x=-Ab/2m$; the graph of x versus t near $t=0$ slopes down.

(c) $a_x=\frac{dv_x}{dt}=Ae^{-(b/2m)t}\left[\left(\frac{b^2}{4m^2}-\omega'^2\right)\cos\omega' t+\frac{\omega' b}{2m}\sin\omega' t\right]$, and at $t=0$, $a_x=A\left(\frac{b^2}{4m^2}-\omega'^2\right)=A\left(\frac{b^2}{2m^2}-\frac{k}{m}\right)$.

(Note that this is $(-bv_0-kx_0)/m$.) This will be negative if $b<\sqrt{2km}$, zero if $b=\sqrt{2km}$ and positive if $b>\sqrt{2km}$. The graph in the three cases will be curved down, not curved, or curved up, respectively.

EVALUATE: $a_x(0)=0$ corresponds to the situation of critical damping.