PH1140 D09 Study Guide One Free, Damped and Forced Oscillations

Preparation. Use this study guide as your "one stop shop" to help you prepare for the first exam. Pay attention in lecture, take notes and highlight important equations. You need these equations for labs and homework. Labs and homework provide basic training and practice for exams, so review these for the exam. Visit the course schedule regularly to ensure that you keep pace with your labs and homework. You may bring one sheet of equations to the exam. Prepare this sheet well.

Key Concepts and Goals. In the first part of the course you study free, damped and forced oscillations of mechanical and electrical systems, a wide class of oscillators described by the same equation of motion

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = \frac{F(t)}{m},$$

the same phasor solutions

$$z(t) = Ae^{-\gamma t/2}e^{i(\omega t+\varphi)} = e^{-\gamma t/2}[C_1 e^{i\omega t} + C_2 e^{-i\omega t}],$$

the same sinusoidal solutions

$$x = Ae^{-\gamma t/2}\cos(\omega t + \varphi) = e^{-\gamma t/2}[B_1\cos(\omega t) + B_2\sin(\omega t)],$$

and similar force and energy equations to those of the mass spring system

$$F = ma = m\ddot{x} = -kx - b\dot{x} + F(t),$$

$$K + U = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = E,$$

differing only in the force and displacement parameters that vary depending on the type of motion (linear, angular, or other).

Free Oscillations. For free mass-spring system, the damping factor $b = 0$ and the applied force $F(t) = 0$. Oscillations are determined by mass m and spring constant k, where $\omega_0^2 = k/m$.

Damped Oscillations. For a damped mass-spring system the applied force $F(t) = 0$. Oscillations are determined by m, k and b, where $\omega_d^2 = \omega_0^2 - \gamma^2/4$ and $\gamma = b/m$.

Forced Oscillations. A forced mass-spring system is subjected to a time-varying force $F(t)$. We often work with the sinusoidal force

$$F(t) = F_{max}\cos\omega t = re\left(F_{max}\, e^{i\omega t}\right).$$

Then the forced solution takes the form (where ω_0 is the free or natural oscillation frequency)

$$x = A\cos(\omega t - \delta) = re\left(Ae^{i(\omega t-\delta)}\right)$$

$$A = \frac{F_{max}/m}{[({\omega_0}^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}, \qquad \tan\delta = \frac{\gamma\omega}{{\omega_0}^2 - \omega^2}.$$

The quality factor $Q = \omega_0/\gamma$ measures the system's ability to resonate with the applied force.

LCR Circuit Oscillations. In the LCR circuit $\omega_0^2 = 1/LC$, and $\gamma = R/L$.

Master these equations and their parameters! Through practice provided by the homework and labs, learn how to analyze an oscillator, identify its force and displacement parameters, classify it as free or damped, linearize it if necessary, and write the corresponding equation of motion and solution. Become fluent with the behavior of, and the relationships between the kinematic quantities $\{x, v, a\}$, the dynamic quantities $\{F, k, b, m, E, K, U\}$, and the oscillator parameters $\{A, \omega, f, T, \gamma, \varphi, Q\}$.

Specific Objectives. Be able to do the following:

1. Define simple harmonic motion, and relate it to circular motion.
2. Derive the equation of motion of an oscillator from its force law.
3. From its equations or description, determine whether an oscillator is free or damped.
4. From its equations or description, determine whether an oscillator is linear or non-linear.
5. Identify the displacement and restoring force from an oscillator's equation of motion.
6. Given displacement and restoring force, write the equation of motion and solution of an oscillator.
7. Given the description an electrical or mechanical oscillator, determine its equation of motion.
8. Apply the $\{A, \varphi\}$ and $\{A_1, A_2\}$ forms of the general solution, either as sinusoids or phasors.
9. Given initial conditions $\{x_o, v_o\}$ determine an oscillator's amplitude and phase.
10. Use the relative phases of kinematic quantities $x, v, a, \text{and } F$ to help analyze system behavior.
11. Use and relate the $\{E, K, U\}$ energy picture of an oscillator to the $\{x, v, a\}$ kinematic picture.
12. Make the small displacement approximation of a non-linear oscillator to obtain a SHO.
13. Analyze forced oscillations and characterize system behavior in terms of amplitude, phase and resonance, quantified by A, δ and Q.