

**1. Membrane Wave.** The wave function given by

$$\zeta(x, z, t) = A \cos(k_x x + k_z z - \omega t + \phi)$$

specifies the transverse displacement of a plane wave moving along a thin elastic membrane of mass density  $\mu = 0.181 \text{ kg/m}^2$ , where

$$A = 2.7 \text{ cm}, \quad \omega = 22\pi \frac{\text{rad}}{\text{s}}, \quad k_x = 30 \frac{\text{rad}}{\text{m}}, \quad k_z = 40 \frac{\text{rad}}{\text{m}}, \quad \phi = \frac{\pi}{12}$$

- Write the wave vector in rectangular and polar form. What is the direction of the wave?
- What are the wave length, wave number and wave speed?
- A wave front is a line of constant phase defined by  $\theta = k_x x + k_z z - \omega t + \phi = \text{constant}$ . At the time  $t = (1/33) \text{ s}$ , graph in the  $\{x, z\}$  plane two consecutive wave fronts defined respectively by the two constant phases  $\theta = 0$  and  $\theta = 2\pi$ . Indicate on the graph the direction of the wave and the wave length. Calculate the distance from the  $\{x, z\}$  origin to the nearest of the two wave fronts, and indicate this distance on the graph.
- What are the maximum transverse displacement, speed and acceleration of a point on the membrane?
- What is the tension per unit length in the membrane?

**2. Ocean Waves.** Tabulate the wave speed  $v(k, h)$ , frequency  $\omega(k, h)$  and displacement  $\eta(k, h)$  of ocean waves for the wave lengths and ocean depths given below. Suggestion: you can work this problem quickly by implementing the formulas given in lecture (also in Study Guide 3) in a Microsoft Excel spreadsheet, where you can utilize the built-in hyperbolic trigonometric functions. In the formula for displacement, use  $A = 1 \text{ m}^2/\text{s}$ .

$\lambda \text{ (m)}$	$h \text{ (m)}$
1	0.1, 1.0, 10, 100
1000	10, 100, 1000, 10000

**3. Challenge Problem for Extra Credit.** A giant wave maker is located at sea where the ocean depth is  $h_{\text{maker}} = 3000 \text{ m}$ , and generates traveling waves of amplitude  $\eta_{\text{max}}(h) = 0.01 \text{ m}$ , at a frequency of  $f = 0.01 \text{ Hz}$ . The wave train reaches a small boat anchored off shore where the depth is  $h_{\text{boat}} = 10 \text{ m}$ . A constant number of wave crests exists between the maker and the boat.

- Do the waves arrive at the boat with the same frequency ( $f_{\text{boat}}$ ) with which they are made ( $f_{\text{maker}}$ )? Hint: if  $f_{\text{maker}} \neq f_{\text{boat}}$ , would the number of wave crests between maker and boat remain constant?
- Determine the wave speed, wave length and wave number of the waves at the maker, and at the boat. Hint: write the formula for frequency  $\omega(k, h)$  in the form

$$\frac{\omega^2 h}{g} = \theta \tanh(\theta), \quad \theta = kh$$

and use a calculator "solve" function to find  $\theta$  in terms of known quantities.