PH1140 B08 **Written Homework 1**

1. A Bobbing Boat Free Oscillator

Swimmers Alice and Beatrice float quietly on a still lake, riding in a small boat with vertical sides, and discuss their weight. Alice is forthcoming with her weight W_1, but Beatrice will not reveal her weight W_2. They continue to sit, and while testing her stopwatch Alice notices that she can make the boat bob up and down with period T. After Beatrice jumps out of the boat to take a swim, Alice finds that the period has changed to T'. When Beatrice returns to the boat, Alice informs her that she knows her weight!

a) Why does it matter that the boat's sides are vertical?
b) Show how Alice was able to determine Beatrice's weight.
c) What one other quantity did Alice need to know in order to make the determination?

Beatrice's friend Cleopatra plans to go boating with Alice. Beatrice advises her that in order to keep her weight secret, she should invite her grandmother Dolores, a non-swimmer who absolutely refuses to reveal her weight.

d) How would this help Cleopatra keep her weight secret?
e) If Dolores is also a swimmer, can Alice determine Cleopatra's weight? Explain.

2. A U-tube Free Oscillator

Knowing that all points on the surface of a still body of water lie at the same level, surveyors Ahmed and Samia check the level of a pyramid by using a long tube filled with water to measure the relative heights of the pyramid's cornerstones. They place the ends of the tube next to adjacent cornerstones, but when they attempt to mark the level of the water at one end of the tube, they see that the water level is slowly oscillating up and down. They decide to average the maximum and minimum levels of the water to obtain the equilibrium level. For this they assume the water oscillates freely.

a) If the length of the water-filled portion of the tube is $L = 111m$, how long will it take to obtain the average?

The surveyors continue to study the water's motion and realize that the motion is decaying, which implies that the average is incorrect.

b) Why is the average incorrect?

Assuming linear damping, the damping coefficient γ may be determined by measuring the levels of three consecutive maxima and timing the period.

c) Show how γ may be determined from these measurements.
d) Given γ, show how to calculate the equilibrium level of the water.

3. A Torsion Oscillator

Plumbers Dennis and Rasmus find a new supplier of copper pipe that offers cost savings, but they are concerned about the purity of the copper and devise a way to test the pipe by measuring the shear modulus n of the pipe metal to an accuracy of 1%. They purchase a length $L = 9.14m$ of pipe having an outside diameter $D = 42.0mm$ and a wall thickness $d = 2.00mm$. They hang the pipe vertically with the top end fixed to the ceiling of their warehouse. They select an iron bar whose moment of inertia about its midpoint is 200 times greater the moment of inertia of the pipe about its axis, $I_{bar} = 1.91\ kg\ m^2 = 200\ I_{pipe}$. They fix the midpoint of the bar to the bottom end of the pipe to create a torsion pendulum. They rotate the iron bar away from its equilibrium position by a small angle in the horizontal plane, then release the bar and time the ensuing torsional oscillations to obtain a period of $T = 0.364\ s$.

a) Why insist that $I_{bar} \gg I_{pipe}$?

b) Determine the shear modulus n of the pipe metal. Is the pipe pure copper?

c) How accurately must I_{bar} be known to ensure an accuracy of $\pm 1\%$ in the determination of n? Are Dennis and Rasmus justified in neglectingI_{pipe} ?

4. Physical Pendulums

You decide to build duck and goose pendulum clocks. You cut duck and goose shapes of equal mass m from a plate of shiny brass. You find the centers of mass (CMs) of the birds and attach their CMs to the ends of identical uniform pendulum rods having length L. You find that the pendulums swing with slightly different periods T_D and T_G. You realize you must offset the birds so that the pendulums will have the same period T, and decide to mount one bird slightly higher on the rod (closer to the pivot) and one bird slighter lower. You recall that the parallel axis theorem applies, and that you can express the measured periods in terms of the moments of inertia I_D and I_G of the duck and goose about the pivot.

a) Do the orientations of the birds matter in determining the offsets? Explain.

b) Given that $T_G > T_D$, determine which moment of inertia, I_D or I_G, is greater.

Challenge:

Assume both $I_{D,CM}$ and $I_{G,CM}$, the moments of inertia of the duck and goose about their respective CMs, are smaller than mL^2. This will be true if L is large compared to the dimensions of the birds.

c) Determine which bird should be mounted higher and which lower.

It is easy to guess the correct answer, but to demonstrate it requires that you apply the given assumption. Without this assumption, the result is ambiguous.

Hints: Use the parallel axis theorem to help you express T_D in terms of $I_{D,CM}$. Introduce a tiny offset by replacing L with $L + \Delta L$. Utilize the binomial expansion in the small quantity $\Delta L/L$

$$\frac{1}{L+\Delta L} = \frac{1}{L(1+\Delta L/L)} = \frac{1}{L}\left(1 - \frac{\Delta L}{L} + \frac{1}{2}\left(\frac{\Delta L}{L}\right)^2 - \cdots\right)$$

and simplify the expression for T_D by neglecting terms involving powers of $\Delta L/L$ greater than one. Finally, collect terms in ΔL, and it should become apparent how the offset $\pm\Delta L$ affects T_D.

5. A Damped Mass-Spring Oscillator

While driving your car, you and a companion marvel at the smooth ride and reflect on the fact that the body of your car (which includes you and your companion, of course) rests on springs and shock absorbers, forming a damped mass-spring oscillator sitting in its equilibrium position. At that very moment you encounter a bump in the road, which causes the car to bounce up and down on its springs.

Immediately after hitting the bump, the initial upward displacement of your "car oscillator" is 9.00 cm. The bouncing motion is quickly damped by the shock absorbers, so that after three complete oscillations, which occur in 4.90 seconds, the amplitude has shrunk to 0.333 cm. Assume that the shock absorbers provide a damping force that is proportional to the rate of change of displacement. When parked, the springs of your car support a weight of 4320 lbs (including you and your companion).

> a) What is the damping coefficient γ?
> b) What is the initial energy of oscillation?
> c) How much energy is lost during the first cycle? During the third cycle?
> d) Where does the energy go?
> e) Tricky question: why do we make the above assumption about the shock absorbers?
> f) Trickier question: where did the energy come from?
> g) In terms of your weight W, what are the maximum and minimum normal forces exerted on you by your seat during the oscillations?
> h) At what times do these occur?

Suppose that to achieve the safest, smoothest ride, your car oscillator should be critically damped.

> i) Is it time to replace your shock absorbers? Explain.
> j) Is the damping affected by the number of passengers in your car? Explain.