PH1140 B08 Written Homework Phasors Due November 6, 2008

1. Consider a phasor z defined by the equation $z = z_1 z_2$, where $z_1 = a + ib,\ z_2 = c + id$.
   a) Show that the length of z is the product of the lengths of z_1 and z_2.
   b) Show that the angle between z and the x axis (that is, the direction of z in the complex plane) is the sum of the angles made by z_1 and z_2 separately.

2. Consider a phasor z defined by the equation $z = z_1/z_2$, with $z_2 \neq 0$, where $z_1 = a + ib,\ z_2 = c + id$.
   a) Show that the length of z is the quotient of the lengths of z_1 and z_2.
   b) Show that the angle between z and the x axis is the difference of the angles made by z_1 and z_2 separately.

3. Show that the multiplication of any phasor z by $e^{i\theta}$ produces a positive rotation of z through the angle θ without any alteration of its length.

4. Find the magnitudes and directions of the phasors $2 + i\sqrt{3}$ and $(2 - i\sqrt{3})^2$.

5. Plot the nine phasors: $z_1 = i$, $z_2 = -i$, $z_3 = e^{i\pi/2}$, $z_4 = e^{-i\pi/2}$, $z_5 = -1$, $z_6 = e^{i\pi}$, $z_7 = e^{-i\pi}$, $z_8 = 1$, and $z_9 = e^{i2\pi}$.

6. Given $z = Ae^{i(\omega t+\varphi)}$, find $\dot{z} = dz/dt$ and $\ddot{z} = d^2z/dt^2$.

7. Given $x = Ae^{-\gamma t}\cos(\omega t + \varphi)$, find $\dot{x} = dx/dt$ and $\ddot{x} = d^2x/dt^2$.

8. Given $z = Ae^{-\gamma t}e^{i(\omega t+\varphi)}$, find $\dot{z} = dz/dt$ and $\ddot{z} = d^2z/dt^2$.

9. Show that $re(z)$, $re(\dot{z})$ and $re(\ddot{z})$ from (8) are equivalent to $x, \dot{x}$ and $\ddot{x}$ from (7).