Practice Problem Set #4: Atoms and Photons I

1. With what mass of nitrogen would 1 g of hydrogen combine to form ammonia, NH_3? (All you need to know is the ratio of the masses of the atoms involved.)

2. Make a Table showing all the stable isotopes for hydrogen, oxygen, and aluminum. Include their percent abundance. You will such information in most chemistry or modern physics textbooks.

3. The mass of a carbon-12 atom is 1.9926465×10^{-26} kg (with an uncertainty of 2 in the last digit). Calculate the value of the atomic mass unit to six significant figures in kg and in MeV/c^2.

4. What is the mass (correct to the nearest atomic mass unit u) of the following molecules: (a) water, H_2O; (b) laughing gas, N_2O; (c) ozone, O_3; (d) glucose, $C_6H_{12}O_6$; (e) ammonia, NH_3; (f) limestone, $CaCO_3$.

5. What is the total charge of 1 mole of Cl^- ions? (The Cl^- ion is a chlorine atom that has acquired one extra electron.) This important quantity is called the *faraday*, and is easily measured. Knowing the electron charge and the *faraday* allows Avogadro's number to be calculated.

6. Given that visible light has $400 < \lambda < 700$ nm range, what is the range of energies of visible photons (in eV)? Ultraviolet (UV) radiation has wavelengths shorter than visible, while infrared (IR) wavelengths are longer than visible. What can you say about the energies of UV and IR photons?

7. (a) X-rays are electromagnetic radiation with wavelengths much shorter than visible – on the order of 1 nm or less. What are the energies of x-ray photons? (b) Electromagnetic waves with wavelengths even shorter than x-rays are called γ-rays (gamma rays) are are produced in many nuclear processes. A typical γ-ray wavelength is 10^{-4} nm (or 100 fm); what is the corresponding photon energy? Given your answer in keV or MeV where appropriate.

8. Microwaves (as used in microwave ovens, telephone transmissions, etc.) are electromagnetic waves with a wavelength on the order of 1 cm. What is the energy of a typical microwave photon in eV?

9. It is found that the radius R of any nucleus is given approximately by $R = R_0A^{1/3}$, where A is the mass number of the nucleus and R_0 is a constant whose value depends a little on how R is defined, but is about $R_0 = 1.1$ fm. (a) What are the radii of the nuclei of helium, carbon,

iron, lead, and lawrencium? (b) How does the volume of a nucleus (assumed to be spherical) depend on A? What does your answer imply about the average density of the nuclei?

10. (a) Calculate the mass of the ^{4}He nucleus to four decimal places in atomic mass units by subtracting the mass of two electrons from that of a neutral ^{4}He atom. (b) The procedure suggested in part (a) is theoretically incorrect because the binding energy of the electrons has been ignored. Given that the binding energy of both electrons (the energy needed to pull both from the helium atom) is 80 eV, is the correct answer bigger or smaller than that given by part (a) and by how much? How many significant figures are needed to see this effect?

11. A lightbulb that is rated at 60 W actually produces only about 3 W of visible light, most of the rest of the energy being heat. (a) About how many visible photons does such a lightbulb produce each second? Use the average wavelength of $\lambda = 550$ nm. (b) If a person looks at such a lightbulb from about 10 ft away, about how many visible photons enter the eye per second? When looking at a bright light, the pupil has a diameter of about 1 mm. (c) By how many powers of 10 does this exceed the minimum detectable intensity of about 100 photons entering the eye per second?

12. The work function for tungsten is $\phi = 4.6$ eV. (a) If light is incident on tungsten, find the critical frequency f_0, below which no electrons will be ejected, and the corresponding wavelength λ_0. Using the energy balance relationship between KE of electron, incoming photon energy, and workfunction, if tungsten is irradiated with light of (b) $\lambda = 200$ nm, and (c) 300 nm, find the maximum KE of the electron.

Solutions #4

PH 1130

1) given 1 g of ^1H, what mass of ^7N needed to form ammonia (NH_3)

$^7N \rightarrow Z = 7$ and $A = 14$ $\therefore m_N = 14\,u$

$^1H \rightarrow Z = 1$ and $A = 1$ $\therefore m_H = 1\,u$

$NH_3 \rightarrow 1N + 3H = 1(14\,u) + 3(1\,u) = 17\,u$

So; ^1H comprises $\frac{3}{17}$ of NH_3 & ^7N comprises $\frac{14}{17}$ of NH_3

$\therefore M_N \cdot \frac{14}{17} + M_H \frac{3}{17} = M_{NH_3}$

$M_{NH_3} = \frac{17}{3} M_H = 5.67\,g$

$M_N = \frac{14}{17} M_{NH_3} = \underline{\underline{4.67}}\,g$

2)

Element	Atomic #	Isotopes (%)
hydrogen (H)	1	^1H (99.985) ^2H (0.015)
helium (He)	2	^3He (0.0001) ^4He (99.9999)
oxygen (O)	8	^{16}O (99.762) ^{17}O (0.038) ^{18}O (0.200)
aluminum (Al)	13	^{27}Al (100)

3) $1\ {}^{12}C$ atom $= 1.992648 \times 10^{-26}$ kg

$1u = \frac{1}{12}\ {}^{12}C = 1.66054 \times 10^{-27}$ kg $= 932.888$ MeV/c^2

4) $H_2O = 2H + 1O = 2(1u) + 1(16u) = 18u$

$N_2O = 2N + 1O = 2(14u) + 1(16u) = 44u$

$O_3 = 3(O) = 3(16u) = 48u$

$C_6H_{12}O_6 = 6(C) + 12(H) + 6(O)$
$= 6(12u) + 12(1u) + 6(16u) = 180u$

$NH_3 = 1N + 3H = 1(14u) + 3(1u) = 17u$

$CaCO_3 = 1Ca + 1C + 3O = 1(40u) + 1(12u) + 3(16u)$
$= 100u$

5) charge of 1 mole of Cl^- ion : $e^- = 1.66 \times 10^{-19}$ C

$= N_A \times 1.66 \times 10^{-19}$ C $= 9.99652 \times 10^4$ C $\simeq 10^5$ C

6) visible light $400 < \lambda < 700$ nm : $E = hf = \frac{hc}{\lambda}$

$$E^- = \frac{hc}{\lambda^-} = \frac{1240 \text{ eV}\cdot\text{nm}}{400 \text{ nm}} = 3.1 \text{ eV}$$

$$E^+ = \frac{hc}{\lambda^+} = \frac{1240 \text{ eV}\cdot\text{nm}}{700 \text{ nm}} = 1.8 \text{ eV}$$

UV light has <u>shorter</u> λ than visible : $\lambda_{UV} < \lambda_V$ $\therefore$ $E_{UV} > E_V$

IR light has <u>longer</u> λ than visible : $\lambda_{IR} > \lambda_V$ $\therefore$ $E_{IR} < E_V$

7) a) $\lambda_{x\text{-ray}} \simeq 0.1$ nm : $E_{x\text{-ray}} \cong \frac{1240 \text{ eV}\cdot\text{nm}}{0.1 \text{ nm}} = 12{,}400 \text{ eV} = 12.4 \text{ keV}$

b) $\lambda_\gamma \cong 10^{-4}$ nm (or 100 fm)

$$E_\gamma \cong \frac{1240 \text{ eV}\cdot\text{nm}}{10^{-4} \text{ nm}} = 1240 \times 10^4 \text{ eV} = 1.240 \times 10^7 \text{ eV}$$

$$= 12.4 \text{ MeV}$$

8) $\lambda_\mu \cong 1$ cm (typical wavelength of microwave radiation)

$\lambda_\mu \simeq 10^{-2}$ m $\simeq 10^{7}$ nm $\therefore$ $E_\mu \simeq \frac{1240 \text{ eV}\cdot\text{nm}}{10^{-7} \text{ nm}} = 1.24 \times 10^{10} \text{ eV}$

$$E_\mu \simeq 12.4 \text{ GeV}$$

9) — radius of nucleus = $R = R_0 A^{1/3}$ $R_0 \simeq 1.1$ fm

a) helium : $A = 4$ $R_{He} = 1.746$ fm
carbon : $A = 12$ $R_C = 2.518$ fm
iron : $A = 56$ $R_{Fe} = 4.208$ fm
lead : $A \cong 208$ $R_{Pb} \cong 6.517$ fm
lawrencium : $A \cong 260$ $R_{Lr} \cong 7.021$ fm

b) $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A \propto A$ linear dependence

Since Volume goes as A and mass ≐ A
then $\rho = \frac{\text{mass}}{\text{volume}} \cong$ constant (nucleus = uniform density)

10) —

a) neutral ^4He atom = 4.0026 u = 6.6483×10^{-27} kg

$m_{e^-} = 9.11 \times 10^{-31}$ kg $2m_{e^-} = 1.822 \times 10^{-30}$ kg

^4He nucleus $= (6.648 \times 10^{-27} - 1.822 \times 10^{-30})$ kg
$= \underline{6.646 \times 10^{-27} \text{ kg} = 4.0013 \text{ u}}$

b) $B = 80$ eV (binding energy of $2e^-$) : $\Delta m = \frac{B}{c^2} = 80 \frac{eV}{c^2}$

$$\Delta m = 80 \frac{eV}{c^2} = 8.59 \times 10^{-8}\ u \qquad \left(1u = 931.5 \frac{MeV}{c^2} = 931.5 \times 10^6 \frac{eV}{c^2} \right)$$

- correct mass in part a) should be <u>smaller</u> by Δm.
- need at least <u>8</u> significant figures.

11) –

60 W total power, 3 W of visible light power
(use $\lambda \approx 550$ nm)

a) # of photons per sec. = N_ν' = photon power / energy of 1 photon

$$N_\nu' = I/E_\nu = (3W)/(hc/\lambda) \ / \ 1.60 \times 10^{-19}\ J/eV$$

$$= \left[\frac{(3\ W)(550\ nm)}{1240\ eV \cdot nm} \right] 6.25 \times 10^{18} \frac{eV}{J} = \underline{8.32 \times 10^{18} \frac{photons}{sec}}$$

b) density of $\frac{photons}{sec}$ $= \frac{N_\nu'}{A} = \rho_\nu = \frac{N_\nu'}{4\pi R^2}$

Area of pupil $= \pi r^2 = \frac{\pi}{4} d^2 = a$

$\therefore$ # of $\frac{photons}{sec}$ thru pupil $= N' = \rho_\nu \cdot a = \frac{N_\nu'}{4\pi R^2} \cdot \frac{\pi d^2}{4} = \frac{N_\nu'}{16} \frac{d^2}{R^2}$

$$\left. \begin{array}{l} d = 1\ mm = 0.1\ cm \\ R = 1\ ft = 30.48\ cm \end{array} \right\} \ \underline{N' = 5.6 \times 10^{12}\ photons/sec}$$

c) min. detectable intensity $\cong$ 100 photons/sec

N' exceeds this by $10^{12}/10^{2} = 10^{10}$ i.e. 10 orders of magnitude

12) —

tungsten work function $= \phi = 4.6$ eV

a) $K_{max} = hf - \phi$: critical freq. $= f_0$ $(\lambda_0 = c/f_0)$

when no e^- are ejected $\therefore$ $hf_0 = \phi$ $(K_{max} = 0)$

$$f_0 = \frac{\phi}{h} = \frac{4.6 \text{ eV}}{4.14 \times 10^{-15} \text{ eV}\cdot\text{s}} = 1.11 \times 10^{15} \text{ s}^{-1} = 1.11 \times 10^{15} \text{ Hz}$$

$$= 1.11 \times 10^{3} \text{ THz}$$

(1110 THz)

$$\lambda_0 = \frac{c}{f_0} = \frac{3 \times 10^{8} \text{ m/s}}{1.11 \times 10^{15} \text{ s}^{-1}} = 2.7 \times 10^{-7} \text{ m} = 0.27\ \mu\text{m}$$

b) $$K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV}\cdot\text{nm}}{200 \text{ nm}} - 4.6 \text{ eV}$$

$$= 1.6 \text{ eV}$$

c) $$K_{max} = \frac{1240 \text{ eV}\cdot\text{nm}}{300 \text{ nm}} - 4.6 \text{ eV} = -0.47 \text{ eV}$$

Since kinetic energy cannot be negative — NO e^- are ejected