Practice Problem Set #3: Relativity III

1. The mass of an electron is about 9.11×10^{-31} kg. Make a table showing the electron's momentum, both the correct relativistic momentum and the classical (non-relativistic) momentum, at speeds with $\beta = 0.1, 0.5, 0.9$, and 0.99.

2. At what speed would a body's relativistic energy E be twice its rest energy of mc^2?

3. (a) Suppose that a mass m has a momentum $\boldsymbol{p}$ and energy E, as measured in a frame S. Use the relations for momentum $\boldsymbol{p} = m(d\boldsymbol{r}/dt_0)$ and energy $E = mc^2(dt/dt_0)$ as well as the known transformation of $d\boldsymbol{r}$ and dt to find the values of $\boldsymbol{p}'$ and E' as measured in another frame S' moving with speed u in the standard configuration. (Remember that dt_0 is the same for all observers) (b) Use the result from part (a) to prove the following: If the total momentum and energy of a system is conserved as measured in one inertial frame S, it must be conserved in any other frame S'.

4. A nuclear particle has a mass of 3 GeV/c^2 and momentum of 4 GeV/c. (a) What is the particle's energy? (b) What is the particle's speed? (Note, 1 GeV = 10^9 eV)

5. When two molecules of hydrogen combine with one molecule of oxygen to form two water molecules, $2H_2 + O_2 \rightarrow 2H_2O$, the energy released is 5 eV. (a) What is the mass difference between the three original molecules and the two final ones? (b) Given that the water molecules has a mass of 3×10^{-26} kg, what is the fractional change in the mass, ΔM/(total mass)? Does it matter significantly whether one uses the initial or final total mass?) (c) If one were to form 10 g of water by this process, what should be the total change in rest mass?

6. Suppose that you fired two, one-gram, putty balls at each other, each with a speed of 10^4 m/s (very fast for putty balls), and that they stick together. (a) What is the mass of the glob after the collision? (b) What is the fractional change in the mass of the putty? (c) How much energy is represented by this change in mass? (d) If it takes 4.2 J of thermal energy to raise the temperature of 1 kg of putty by 1 °C, what is the temperature increase associated with the energy change?

7. A subatomic particle A decays into two identical particles B, $A \rightarrow B + B$. The two B particles are observed to have exactly equal and opposite momenta of magnitude p. (a) What can you deduce about the velocity of A just before the decay? (b) Derive an expression for the mass m_A, of A in terms of m_B and p.

8. A lambda particle (Λ) decays into a proton and a pion, $\Lambda \rightarrow p + \pi$, and is observed that the proton is left at rest. (a) What is the energy of the pion? (b) What is the energy of the original Λ? Note that the rest masses are $m_\Lambda = 1116$, $m_p = 938$, and $m_\pi = 140$ all in MeV/c^2.

9. The K^o meson is a subatomic particle of rest mass $m_K = 498$ MeV/c^2 that decays into two charged pions, $K^o \rightarrow \pi^+ + \pi^-$. (The π^+ and π^- have opposite charges but exactly the same mass of $m_\pi = 140$ MeV/c^2) A K^o at rest decays into two pions. Use conservation of energy and momentum to find the energies, momenta, and speeds of the two pions.

Solutions #3 PH1130

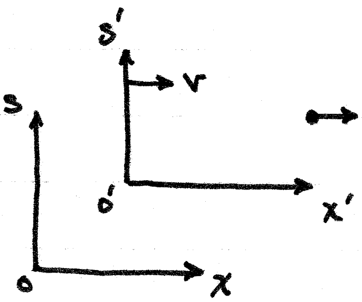
1) $m_e = 9.11 \times 10^{-31}$ kg , $P_{class.} = mv$, $P = \gamma mv$

β	P_{class}	P	γ
0.1	2.733×10^{-23}	2.747×10^{-23}	1.005038
0.5	1.3665×10^{-22}	1.578×10^{-22}	1.154701
0.9	2.460×10^{-22}	5.643×10^{-22}	2.294157
0.99	2.706×10^{-22}	1.918×10^{-21}	7.088812

2) what speed for $E = 2mc^2 = \gamma mc^2$?

$\therefore \quad \gamma = 2 = \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2}$ So; $\underline{u = 0.866\,c}$

3)



$\longrightarrow \vec{p}\,,\, E$ in S

$$\vec{p} = m\frac{d\vec{r}}{dr_o} = m\frac{\Delta\vec{r}}{\Delta r_o}$$

$$E = mc^2\frac{\Delta t}{\Delta t_o}$$

$\Delta r = \gamma(\Delta r' + \beta c\Delta t')$ or $\Delta r' = \gamma(\Delta r \mp \beta c \Delta t)$

$\Delta t = \gamma(\Delta t' + \frac{\beta}{c}\Delta r')$ or $\Delta t' = \gamma(\Delta t - \frac{\beta}{c}\Delta r)$

Note : $\frac{p\,\Delta r_o}{m} = \Delta r$ and $\frac{E\,\Delta t_o}{mc^2} = \Delta t$

Substitute and Solve for p & E.

4) $m = 3 \frac{GeV}{c^2}$ and $p = 4 \frac{GeV}{c}$ $(1 \text{ GeV} = 10^9 \text{ eV})$

a) $E^2 = (pc)^2 + (mc^2)^2$

$= \left(4 \frac{GeV}{\cancel{c}} \cancel{c}\right)^2 + \left(3 \frac{GeV}{\cancel{c^2}} \cancel{c^2}\right)^2 = 16 \text{ GeV}^2 + 9 \text{ GeV}^2$

$= 25 \text{ GeV}^2$

$\therefore$ $\underline{E = 5 \text{ GeV}}$

b) $E = \gamma mc^2 = 5 \text{ GeV} = \gamma \, 3 \frac{GeV}{\cancel{c^2}} \cancel{c^2}$ $\therefore$ $\gamma = 1.667$

$\gamma = (1-\beta^2)^{-1/2}$, $\gamma^{-2} = 1-\beta^2$, $\beta^2 = 1-\gamma^{-2}$

$\beta = (1-\gamma^{-2})^{1/2}$

$\therefore$ $u = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} c = \underline{0.8\,c}$

5) $2H_2 + O_2 \rightarrow 2H_2O$ energy released $= \Delta E = 5 \text{ eV}$

a) $\Delta M = 2m_{H_2O} - (2m_{H_2} + m_{O_2}) = \frac{\Delta E}{c^2} = 5 \text{ eV}/c^2$

$= 5 \frac{(1.6 \times 10^{-19} \text{ J})}{(3 \times 10^8 \text{ m/s})^2} \cong 9 \times 10^{-36} \text{ kg}$

b) $\frac{\Delta M}{2m_{H_2O}} = \frac{9 \times 10^{-36}}{3 \times 10^{-26}} \cong 3 \times 10^{-10}$ doesn't matter which total mass used.

c) total of 10 g of H_2O : $\Delta M = \left(\frac{\Delta M}{2m_{H_2O}}\right) 0.005 \text{ kg} = \underline{15 \times 10^{-13} \text{ kg}}$

6) before $\quad m_1, v_1 \quad O \rightarrow \qquad \leftarrow O \quad m_2, v_2$

after $\quad \odot \quad m_T \quad v_T$

$m_1 = m_2 = 1\,g = 0.001\ kg$

$v_1 = v_2 = 10^4\ m/s$

$$\beta_1 = \beta_2 = \frac{10^4}{3 \times 10^8} \cong 0.33 \times 10^{-4}$$

$$\gamma_1 = \gamma_2 = 1.0000000001$$

momentum: $\quad P_1 + P_2 = P_T = 0$

$$\therefore \quad v_T = 0$$

energy: $\quad E_1 + E_2 = E_T \quad \longrightarrow \quad \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_T c^2$

a) $m_T = \gamma_1 m_1 + \gamma_2 m_2 = 2\gamma_1 m_1 = 2.0000000002 \times 10^{-3}\ kg$

b) $\dfrac{m_T - (m_1 + m_2)}{m_1 + m_2} \cong 5.6 \times 10^{-10}$

c) $\Delta M = m_T - (m_1 + m_2) \cong 1.12 \times 10^{-12}\ kg \quad \therefore \ \Delta E = \Delta M c^2 \cong 1.01 \times 10^{5}\ J$

d) 4.2 J to raise 1 kg of putty 1 °C : $\quad Q = 4.2\ \dfrac{J}{kg\,°C}$

$$\Delta T = \frac{\Delta E}{m_T Q} \cong 1.2 \times 10^{7}\ °C$$

- very large amounts of energy from a very small amount of mass change.

7) $-$ $A \rightarrow B + B$ with B-momentum equal but opposite direction

$$p^i = p^f \quad \text{\&} \quad E^i = E^f$$

a) $P_B - P_B = 0 = P_A = \gamma_A m_A v_A \quad \therefore \quad v_A = 0$ So, particle A was <u>stationary</u>

b) $$m_A c^2 = \gamma_B m_B c^2 + \gamma_B m_B c^2 = 2\gamma_B m_B c^2$$

$$= 2\sqrt{(m_B c^2)^2 + (P_B c)^2} \quad \therefore \quad \boxed{m_A = \frac{2}{c^2}\sqrt{(m_B c^2)^2 + (P_B c)^2}}$$

Note: in terms of m_B & P_B

8) $-$ $\Lambda \rightarrow p + \pi$ where proton is found at <u>rest</u>

$$m_\Lambda = 1116 \ \frac{\text{MeV}}{c^2}, \quad m_p = 938 \ \frac{\text{MeV}}{c^2}, \quad m_\pi = 140 \ \frac{\text{MeV}}{c^2}$$

a) $$E_\Lambda = E_p + E_\pi \rightarrow \sqrt{(P_\Lambda c)^2 + (m_\Lambda c^2)^2} = m_p c^2 + \sqrt{(P_\pi c)^2 + (m_\pi c^2)^2}$$

$$(P_\Lambda c)^2 + (m_\Lambda c^2)^2 = (m_p c^2)^2 + (P_\pi c)^2 + (m_\pi c^2)^2 + 2m_p c^2 \underbrace{\sqrt{(P_\pi c)^2 + (m_\pi c^2)^2}}_{E_\pi}$$

$P_\Lambda = P_p + P_\pi$ but $P_p = 0 \quad \therefore \quad P_\Lambda = P_\pi$

So, $$(m_\Lambda c^2)^2 = (m_p c^2)^2 + (m_\pi c^2)^2 + 2m_p c^2 E_\pi$$

So, $$E_\pi = \frac{(m_\Lambda c^2)^2 - \left[(m_p c^2)^2 + (m_p c^2)^2\right]}{2\, m_p c^2}$$

$$= \frac{(1116)^2 - \left[(938)^2 + (140)^2\right]}{2\,(938)} \ \text{MeV}$$

$$\underline{E_\pi = 184.4 \ \text{MeV}}$$

b) $E_\Lambda = E_p + E_\pi = (938) + (184.4) = \underline{1122.4 \ \text{MeV}}$

9) — $K^\circ \to \pi^+ + \pi^-$ $\quad m_K = 498 \ \frac{\text{MeV}}{c^2}$

$\underline{K^\circ \text{ @ rest}}$ $\quad m_\pi = 140 \ \text{MeV}/c^2$

$$E^i = m_K c^2 = E^f = E_{\pi^+} + E_{\pi^-} = 2 E_\pi$$

$$p^i = p^f \rightarrow 0 = P_\pi + P_\pi \ \therefore \ P_{\pi^+} = -P_{\pi^-} \ \ \& \ \ (P_\pi c)^2 = (P_{\pi^-} c)^2$$

Now; $E_\pi = \frac{1}{2} E_K = \frac{1}{2} m_K c^2 = \frac{1}{2}(498)\ \text{MeV} = 249 \ \text{MeV} = E_{\pi^\mp}$

$$P_{\pi^+} = \frac{1}{c}\sqrt{E_\pi^2 - (m_\pi c^2)^2} = 205.9 \ \frac{\text{MeV}}{c} = -P_{\pi^-}$$

$$P_\pi = \gamma_\pi m_\pi V_\pi \ \therefore \ V_\pi = \left(\frac{P_\pi}{m_\pi}\right)\left[c^2 + \left(\frac{P_\pi}{m_\pi}\right)^2\right]^{-1/2} c$$

$$= 0.827\, c = V_{\pi^+}$$

$$V_{\pi^-} = -0.827\, c$$