Practice Problem Set #2: Relativity II

1. The two frames S and S' are in the standard configuration (origins coinciding at $t = t' = 0$, x and x' axes parallel, and the relative velocity along the x-axis). Their relative speed is $0.5c$. An event occurs on the x-axis at $x = 10$ light seconds (that is $x = 3\times10^9$ m, the distance light would travel in 10 seconds) at a time $t = 4$ s in the frame S. What are its coordinates x', y', z', t' as measured in S'?

2. The Lorentz transformation consists of four equations giving x', y', z', t' in terms of x, y, z, t. Solve these equations to give x, y, z, t in terms of x', y', z', t'. Show that you get the same result by interchanging primed and unprimed variables and replacing u with $-u$.

3. A traveler in a rocket of length $2d$ sets up a coordinate system S' with origin O' anchored at the exact middle of the rocket and the x'-axis along the rocket's length. At time $t' = 0$ the traveler ignites a flashbulb at O'. (a) Write down the coordinates x_F', t_F' and x_B', t_B' for the arrival of the light at the front (F) and back (B) of the rocket. (b) Now consider the same experiment as observed in a frame S relative to which the rocket is traveling at a speed u (S and S' are in the standard configuration). Use the Lorentz transformation to find the coordinates x_F, t_F and x_B, t_B of the arrival of the two signals. Explain why the two times are not equal in frame S, although they were equal in S'.

4. A rocket (rest frame S') traveling at speed $u = 0.5$ c relative to the earth (rest frame S) shoots forward a bullet traveling at speed $v' = 0.6$ c relative to the rocket. What is the bullet's speed relative to the earth?

5. Muons are sub-atomic particles that are produced several kilometers above the earth's surface due to the collision of cosmic rays (charged particles, such as protons, that enter the earth's atmosphere from space) with atoms in the atmosphere. These muons rain down on the earth more-or-less uniformly, although some decay on the way down since the muon is unstable and has a proper half-life of 1.5 μs (10^{-6} s). In a certain experiment, a muon detector is carried on a balloon to an altitude of 2000 m, and in the course of 1 hour registers 650 muons traveling at 0.99 c toward the earth. If an identical detector is at sea level (ground), how many muons will it register in 1 hour? Note that if N_0 is the starting population of muons, then after n half-lives, $N = N_0/2^n$ muons remain. (a) What is the pathlength in the muon frame? (b) What is the pathlength in the earth frame? (c) Which pathlength is proper? (d) Which frame would give the proper time of flight? (e) Determine the interval between the balloon and earth in the muon and earth frame of reference. Are they the same?

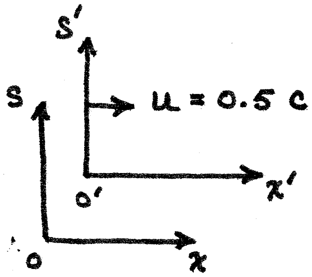
6. In a frame S, two events have spatial separation of $\Delta x = 600$ m, $\Delta y = \Delta z = 0$, and temporal separation $\Delta t = 1$ μs. A second frame S' is in the standard configuration moving at a speed u. In S', the spatial separation is also found to be $\Delta x' = 600$m. What is u and $\Delta t'$?

7. Observers in a frame S arrange for two simultaneous explosions at time $t = 0$. The first explosion is at the origin ($x_1 = y_1 = z_1 = 0$) while the second explosion is on the positive x-axis 4 light-years away ($x_2 = 4 \cdot c$, $y_2 = z_2 = 0$). (a) Use the Lorentz transformation to find the coordinates of these two events as observed in a frame S' traveling in the standard configuration moving at a speed 0.6 c relative to S. (b) How far apart are the two events as measured in S' ? (c) Are the events simultaneous in S' ?

8. As seen from earth (rest frame S) two rockets A and B are approaching each other in opposite directions, each with a speed 0.9 c relative to S. Find the velocity of rocket B as measured by an observer on rocket A.

Solutions #2 PH 1130

1)



origins O & O' coincident @ $t = t' = 0$

$\beta = 0.5$ & $\gamma = 1.15$

event ocurs @ $x = 10\,c\cdot s = 10\,s \times 3\times10^8\,m/s = 3\times10^9\,m$
$t = 4s$

in S

$x = 3\times10^9\,m$

$y = z = 0$

$t = 4s$

in S'

$x' = \gamma(x - \beta c t) = 9.2\,cs = 2.76\times10^9\,m$

$y' = z' = 0$

$t' = \gamma(t - \beta\frac{x}{c}) = -1.15\,s$

2) 1. $x' = \gamma(x - \beta c t)$

2. $y' = y$

3. $z' = z$

$\Big\}$ trivial

4. $t' = \gamma(t - \beta\frac{x}{c})$

1. $x = \frac{x'}{\gamma} + \beta c t$ & 2. $t = \frac{t'}{\gamma} + \frac{\beta x}{c}$

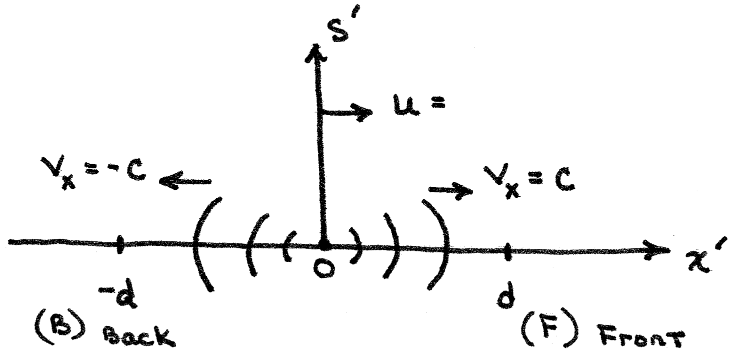
$\therefore \quad x = \frac{x'}{\gamma} + \beta c\left(\frac{t'}{\gamma} + \frac{\beta x}{c}\right) = \frac{x'}{\gamma} + \frac{\beta c t'}{\gamma} + \beta^2 x$

So, $x(1-\beta^2) = \frac{1}{\gamma}(x' + \beta c t') \quad \therefore \quad x = \gamma(x' + \beta c t')$

$\therefore \quad t = \frac{t'}{\gamma} + \frac{\beta}{c}\left(\frac{x'}{\gamma} + \beta c t\right) = \frac{t'}{\gamma} + \frac{\beta x'}{c\gamma} + \beta^2 t$

So, $t(1-\beta^2) = \frac{1}{\gamma}(t' + \frac{\beta x'}{c}) \quad \therefore \quad t = \gamma(t' + \beta\frac{x'}{c})$

3)



flash @ $t' = 0$

Note : $v = \frac{\Delta s}{\Delta t}$

a) $x'_B = -d$ $\qquad$ $x'_F = d$

$t'_B = \frac{d}{c}$ $\qquad$ $t'_F = \frac{d}{c}$

b) $x_B = \gamma(x'_B + \beta c t'_B)$ $\qquad$ same for x_F and t_F

$t_B = \gamma(t'_B + \frac{\beta x'_B}{c})$

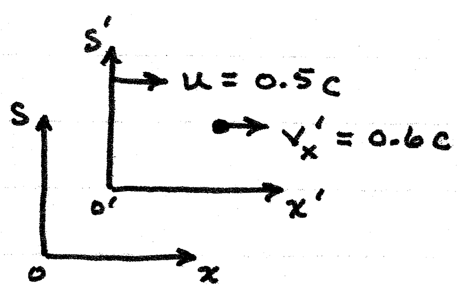
$$x_B = \gamma(-d + \beta c \frac{d}{c}) = \gamma d (\frac{u}{c} - 1)$$

$$t_B = \gamma(\frac{d}{c} - \beta \frac{d}{c}) = \gamma \frac{d}{c}(1 - \frac{u}{c})$$

$$x_F = \gamma(d + \beta c \frac{d}{c}) = \gamma d (1 + \frac{u}{c})$$

$$t_F = \gamma(\frac{d}{c} + \beta \frac{d}{c}) = \gamma \frac{d}{c}(1 + \frac{u}{c})$$

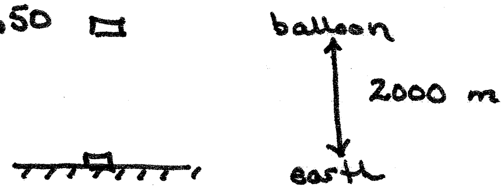
4)



$\beta = \frac{u}{c} = 0.5$

$$v_x = \frac{v'_x + u}{1 + \frac{\beta}{c} v'_x} = \frac{0.6c + 0.5c}{1 + \frac{0.5}{\cancel{c}}(0.6\cancel{c})} = \frac{1.1}{1.3} c = 0.85c$$

5) $\frac{N_i}{hr} = 650$



muons traveling @ $u = 0.99\,c$

proper half-life $= 1.5\,\mu s$

Note: $\beta = 0.99$ and $\gamma = 7.089$

a) pathlength in muon frame : pathlength @ rest in earth frame

$\therefore$ proper pathlength $= \ell^\circ = 2000\,m$

$$\ell = \frac{\ell^\circ}{\gamma} = 282.1\ m$$

b) proper pathlength $= 2000\ m$

c) earth frame = proper frame for path length

d) time of flight (earth-frame) $= \Delta t = \frac{\ell^\circ}{u} = 6.734\ \mu s$

$\Delta t'$ (muon frame) $= \frac{\ell}{u} = 0.95\ \mu s$ = shortest time $\therefore$ muon frame = proper frame for time of flight

e) $(\Delta\tau)^2 = (c\,\Delta t)^2 - (\ell^\circ)^2 \cong 7.7 \times 10^4\ m^2 \rightarrow$ time-like

$(\Delta\tau')^2 = (c\,\Delta t')^2 - (\ell)^2 \cong -7.7 \times 10^4\ m^2 \rightarrow$ space-like

Comment : Same value but opposite in sign

Now, $t_{1/2} = \gamma\, t^\circ_{1/2} = 10.633\ \mu s$ half-life in earth frame $\therefore\ n = \frac{\Delta t}{t_{1/2}} = 0.6333$

$$N_f/hr = N_i/hr \left(\tfrac{1}{2}\right)^n = 419\,/\,hr$$

Note: $n = \frac{\Delta t'}{t^\circ_{1/2}} = 0.6333$

6) 2 events seperated by $\left.\begin{array}{l}\Delta x = 600\ m \\ \Delta y = \Delta z = 0 \\ \Delta t = 1\ \mu s\end{array}\right\} S$

in S' ; $\Delta x' = 600\ m$

$\Delta y' = \Delta z' = 0$ Find u and $\Delta t'$.

$$\Delta x' = \gamma(\Delta x - \beta c \Delta t) = \gamma(\Delta x - u\Delta t)$$

$$(1-\beta^2)\,\Delta x'^2 = (\Delta x - u\Delta t)^2 = \Delta x^2 + u^2\Delta t^2 - 2u\Delta t\Delta x$$

$$(c^2 - u^2)\,\Delta x'^2 = c^2\Delta x^2 + c^2u^2\Delta t^2 - 2uc^2\Delta t\Delta x$$

$$-u^2\Delta x'^2 = c^2u^2\Delta t^2 - 2uc^2\Delta t\Delta x$$

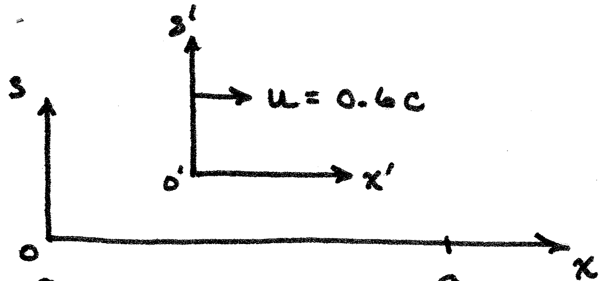
$$u^2(c^2\Delta t^2 - \Delta x'^2) + 2uc^2\Delta t\Delta x = 0$$

$$u\left[u(c^2\Delta t^2 - \Delta x'^2) + 2c^2\Delta t\Delta x\right] = 0$$

2 solutions : $u = 0$ & $u = \dfrac{-2c^2\Delta t\Delta x}{(c^2\Delta t^2 - \Delta x'^2)} = 0.8c$

From $\Delta t' = \gamma\left(\Delta t - \dfrac{u\Delta x}{c^2}\right) = -1\ \mu s$

7)



$\beta = 0.6$ & $\gamma = 1.25$

①	②
$t_1 = 0$	$t_2 = 0$
$x_1 = 0$	$x_2 = 4$ c-yr

a), b), c)

$$x_1' = \gamma(x_1 - \beta c t_1) = 0$$

$$x_2' = \gamma(x_2 - \beta c t_2) = \gamma x_2 = 5 \text{ c·yr}$$

$$t_1' = \gamma\left(t_1 - \beta \frac{x_1}{c}\right) = 0$$

$$t_2' = \gamma\left(t_2 - \beta \frac{x_2}{c}\right) = -\gamma \beta \frac{x_2}{c} = -3 \text{ yr}$$

$\Delta x = x_2 - x_1 = 4$ c·yr $\qquad \Delta t = 0$

$\Delta x' = x_2' - x_1' = 5$ c·yr $\qquad \Delta t' = t_2' - t_1' = -3$ yr

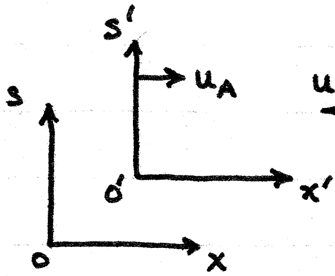
8.)

Ⓐ ▷→ 0.9c $\qquad$ 0.9c ←◁ Ⓑ

$u_A = +0.9c$

$u_B = -0.9c$

Ⓔ



$u_B = V_B$ ←

$\beta = 0.9$

$$V_B' = \frac{V_B - u_A}{\left(1 - \frac{\beta V_B}{c}\right)} = \frac{-1.8}{1 + 0.81} c = -0.994\,c$$

$$V_A'' = \frac{V_A + u_B}{1 + \beta \frac{V_A}{c}} = \frac{1.8}{1 + 0.81} c = +0.994\,c$$