Practice Problem Set #1: Relativity I

1. (a) What must one's speed be, relative to S, in order that one's clocks will lose 1 second per day as observed by S'? (b) What if they lose 1 minute per day?

2. A space explorer sets off at a steady $v = 0.95c$ to a distant star. After exploring the star for a short time, the explorer returns at the same speed and arrives home after a total absence of 80 years (as measured by someone at home). How long do the explorer's clocks say the trip took? How much has the explorer aged?

3. A rocket of proper length 40 m is observed to be 32 m long as it passes the observer. What is the rocket's speed relative to the observer?

4. A meter-stick is moving with a speed of $0.8c$ relative to frame S. (a) What is the stick's length as measured by an observer in frame S, if the stick is parallel to its velocity $\mathbf{v}$? (b) What if the stick is perpendicular to $\mathbf{v}$? (c) What if the stick is at 60° to $\mathbf{v}$, as seen in the stick's rest frame? (d) What if the stick is at 60° to $\mathbf{v}$, as measured in frame S?

5. The two frames S and S' are in the standard configuration (origins coinciding at $t = t' = 0$, x and x' axes parallel, and the relative velocity along the x-axis). Their relative speed is $0.5c$. An event occurs on the x-axis at $x = 10$ light seconds (that is $x = 3\times10^9$ m, the distance light would travel in 10 seconds) at a time $t = 4$ s in the frame S. What are its coordinates x', y', z', t' as measured in S'?

6. Two inertial frames S and S' are in the standard configuration, with relative velocity v along the line of the x and x' axes. Consider any two events, 1 and 2. (a) From the Lorentz transformations, derive expressions for the separations $\Delta x'$, $\Delta y'$, $\Delta z'$, $\Delta t'$ in terms of Δx, Δy, Δz, Δt between these two events. (b) If $\Delta x = 0$ and $\Delta t = 4$ s, whereas $\Delta t' = 5$ s, what is the relative speed v and what is $\Delta x'$?

7. The frames S and S' are in the standard configuration with relative velocity $0.8c$ along the x-axis. (a) What are the coordinates (x_1, y_1, z_1, t_1) in S of an event that occurs on the x'-axis with $x_1' = 1500$ m and $t_1' = 5$ μs? (b) Answer the same for a second event on the x'-axis with $x_2' = -1500$ m and $t_2' = 10$ μs. (c) What are the time intervals, Δt and $\Delta t'$, between the two events as measured in S and S'?

8. (a) Two events occur in an inertial frame: event 1 occurs at the origin at time $t_1 = 0$ s, and event 2 occurs at $(x_2, y_2, z_2) = (4, 0, 0)$ m at time $t_2 = 1\text{x}10^{-8}$ s. Can you find another inertial frame in which event 2 precedes event 1? If so, give an example of such a frame, and calculate the coordinates and times of the two events in the new frame.

9. A group of π mesons is observed traveling at a speed of $0.8c$ in a particle-physics laboratory. (a) What is the factor γ for the pions (π-mesons)? (b) If the pions' proper half-life is 1.8×10^{-8} s, what is their half-life as observed in the lab frame? (c) If there were initially 32,000 pions, how many will be left after they travel 36 m? (d) What would the answer to (c) be if one ignored time dilation?

10. In a laboratory, a proton collides with a hydrogen atom at $(x, y, z) = (0, 0, 0)$ meters. Simultaneously, a pion decays into a muon and a neutrino at $(x, y, z) = (1000, 0, 0)$ meters. Calculate the separation in time between these two events in a frame in which the two events occur at $(x', y', z') = (0, 0, 0)$ and $(125, 0, 0)$ meters, respectively. Comment on the meaning of "simultaneous".

Solutions #1 PH 1130

1. a) what u wrt/ S in order to lose 1 s/day wrt/ S' ?

Note : $\Delta t' = 1 \text{ day} = 24 \text{ hr} = 1440 \text{ min} = 86400 \text{ s}$

$\Delta t = 1 \text{ day} \pm 1 \text{ sec} = 86401 \text{ s}$

time dilation - $\Delta t = \Delta t' \gamma$, $\gamma = (1-\beta^2)^{-1/2}$

Solve for u : $u = \left[1 - \left(\frac{\Delta t'}{\Delta t}\right)^2\right]^{1/2} c = 0.00340\, c$

b) what u wrt/ S in order to lose 1 min/day wrt/ S' ?

Now : $\Delta t' = 1 \text{ day} = 86400 \text{ s}$

$\Delta t = 1 \text{ day} + 1 \text{ min} = 86460 \text{ s}$

So, from a) $u = \left[1 - \left(\frac{\Delta t'}{\Delta t}\right)^2\right]^{1/2} c = 0.0372\, c$

2. $u = 0.95\, c$ out and back ; 80 yr round trip wrt/ earth $= \Delta t$

what is $\Delta t'$ (wrt/ rocket) ?

Note : $\Delta t' = \frac{\Delta t_{1/2}}{\gamma} + \frac{\Delta t_{1/2}}{\gamma} = \frac{1}{\gamma}(\Delta t_{1/2} + \Delta t_{1/2}) = \frac{\Delta t}{\gamma}$

$\Delta t' = \Delta t\,(1-\beta^2)^{1/2} = 24.98 \text{ yr}$ (ignoring any time spent at planet or star)

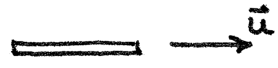
3. proper length = 40 m wrt/ S′ , observed length = 32 m wrt/ S , what is u wrt/S ?

$l_0 = 40$ m and $l = 32$ m

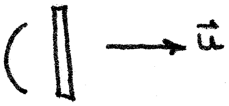
length contraction - $l = \dfrac{l_0}{\gamma} = l_0 (1-\beta^2)^{1/2}$

Solve for u : $u = \left[1 - \left(\dfrac{l}{l_0}\right)^2\right]^{1/2} c = 0.6\,c$

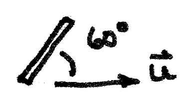
4. meterstick ($l_0 = 1$ m) moves with u wrt/ S : what is l if ...

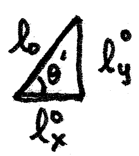
a) meterstick parallel to $\vec{u}$ () - $u = 0.8c$

$l = \dfrac{l_0}{\gamma} = l_0 (1-\beta^2)^{1/2} = 0.6$ m

b) meter stick perpendicular to $\vec{u}$ () - $u = 0.8c$

ignoring the meter stick width, $l = l_0 = 1$ m

c) meterstick 60° to $\vec{u}$ in S′ () - $u = 0.8c$

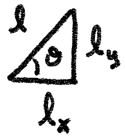


in S′ : $l_y^0 = l_0 \sin\theta'$ ← not contracted

$l_x^0 = l_0 \cos\theta'$ ← contracted

$l = (l_x^2 + l_y^2)^{1/2} = l_0 \left(\dfrac{\cos^2\theta'}{\gamma} + \sin^2\theta'\right)^{1/2} = 0.917$ m

d) meterstick 60° to $\vec{u}$ in S ($\xrightarrow{60^\circ} \vec{u}$) - $u = 0.8c$



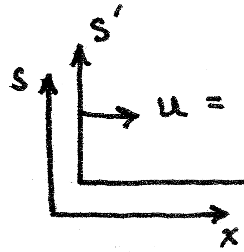
in S : $\left.\begin{array}{l} l_y = l \sin\theta \\ l_x = l \cos\theta \end{array}\right\}$ $\theta = \tan^{-1}\left(\frac{l_y}{l_x}\right) = 60^\circ$

from c) $l_y = l_y^\circ \sin\theta'$ and $l_x = l_x^\circ \frac{\cos\theta'}{\gamma}$

$$\tan 60^\circ = \frac{l_y}{l_x} = (\tan\theta')\,\gamma \quad \therefore\ \theta' = \tan^{-1}\left(\frac{\tan 60^\circ}{\gamma}\right) = 46^\circ$$

So; $$l = l_o\left(\frac{\cos^2\theta'}{\gamma} + \sin^2\theta'\right)^{1/2} = 0.899 \text{ m}$$

5. Standard Configuration :



$u = 0.5c = 1.5 \times 10^8$ m/s

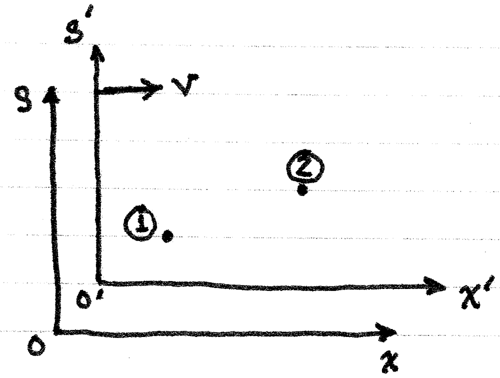
origins coincident at $t = t' = 0$

event occurs at $x = 10$ light sec $= 3 \times 10^9$ m at $t = 4$ s in S.
what are the event's coordinates in S' ?

Lorentz transforms — Standard Configuration

$$\begin{aligned}
x' &= \gamma(x - ut) &&= 2.77 \times 10^9 \text{ m} \\
y' &= y &&= 0 \text{ m} \\
z' &= z' &&= 0 \text{ m} \\
t' &= \gamma\left(t - \frac{ux}{c^2}\right) &&= -1.155 \text{ s}
\end{aligned}$$

6. S & S' in Standard Config.



Eq 2.26
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Consider any two events 1 and 2

Eq. 2.27
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

a) $\Delta x' = x_2' - x_1' = \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1) = \gamma(\Delta x - v\Delta t)$

$\Delta y' = y_2' - y_1' = y_2 - y_1 = \Delta y$

$\Delta z' = \Delta z$

$\Delta t' = t_2' - t_1' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right) - \gamma\left(t_1 - \frac{vx_1}{c^2}\right) = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$

b) $\Delta x = 0$ and $\Delta t = 4\,s$ whereas $\Delta t' = 5\,s$; find v and $\Delta x'$

from $\Delta t = 4\,s$ and $\Delta t' = 5\,s$: $\Delta t' = \gamma\left(\Delta t - \cancel{\frac{v\Delta x}{c^2}}^{\,0}\right) = \gamma\Delta t$

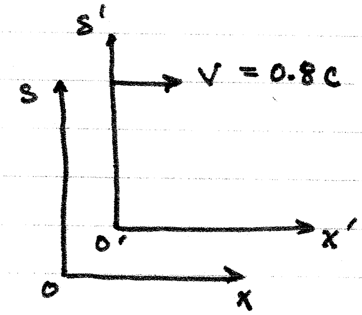
$\gamma = \frac{\Delta t'}{\Delta t} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ $\therefore$ $\left(\frac{\Delta t}{\Delta t'}\right)^2 = 1 - \frac{v^2}{c^2}$

$$v^2 = \left(1 - \left(\frac{\Delta t}{\Delta t'}\right)^2\right)c^2$$
$$= \sqrt{1 - \left(\frac{\Delta t}{\Delta t'}\right)^2}\, c$$

$\gamma = 1.25$ and $\boxed{v = 0.6\,c}$

$$\Delta x' = \gamma(\cancel{\Delta x}^{\,0} - v\Delta t) = -v\gamma\Delta t = -9\times 10^{8}\,m$$

7.



$\beta = 0.8$

$\gamma = 1.667$

a) $x_1' = 1500\,m$ $\quad x_1 = \gamma(x_1' + vt_1') = 1.667(1500 + 0.8c(5\times10^{-6}))$

$= 4501\,m$

$y_1' = 0 \quad y_1 = 0$

$z_1' = 0 \quad z_1 = 0$

$t_1' = 5\mu s \quad t_1 = \gamma\left(t_1' + \frac{vx'}{c^2}\right) = 1.667\left(5\times10^{-6} + \frac{0.8(1500m)}{c}\right)$

$= 15\times10^{-6}\,s$

$= 15\,\mu s$

b) $x_2' = -1500\,m$ $\quad x_2 = 1.667(-1500 + 0.8c\ 10\times10^{-6}) = 1500\,m$

$y_2' = 0 \quad y_2 = 0$

$z_2' = 0 \quad z_2 = 0$

$t_2' = 10\mu s \quad t_2 = 1.667\left(10\times10^{-6} + \frac{0.8(-1500)}{c}\right) = 10\times10^{-6}\,s$

$= 10\,\mu s$

c) $$\left.\begin{aligned} \Delta t &= t_2 - t_1 = 10\,\mu s - 15\,\mu s = -5\,\mu s \\ \Delta t' &= t_2' - t_1' = 10\,\mu s - 5\,\mu s = 5\,\mu s \end{aligned}\right\}$$

8. a) event 1 = $(x_1, y_1, z_1) = (0,0,0)$ @ $t_1 = 0\,s$

event 2 = $(x_2, y_2, z_2) = (4,0,0)\,m$ @ $t_2 = 1 \times 10^{-8}\,s$

$\Delta t = t_2 - t_1 = 1 \times 10^{-8}\,s$ $\quad \Delta t' = t_2' - t_1' = \gamma\left(\Delta t - \frac{u}{c^2}\Delta x\right)$

$\Delta x = x_2 - x_1 = 4\,m$ $\quad \Delta x' = x_2' - x_1' = \gamma(\Delta x - u\Delta t)$

for event 2 to precede event 1 : $\Delta t' < 0$

$\therefore \quad \frac{u}{c^2}\Delta x > \Delta t$ or $\boxed{u > c^2\left(\frac{\Delta t}{\Delta x}\right) > \frac{3}{4}c}$

b) event 3 = $(x_3, y_3, z_3) = (0,0,0)$ @ $t_3 = 0\,s$

event 4 = $(x_4, y_4, z_4) = (4,0,0)\,m$ @ $t_4 = 2 \times 10^{-8}\,s$

following part a) $\frac{u\,\Delta x}{c^2} > \Delta t$ or $u > c^2 \frac{\Delta t}{\Delta x}$

using these values : event 4 precedes 3 if $u > 1.5\,c$

Since $u > c$ therefor <u>NO</u> other frame allows 4 to precede 3.

9. : π mesons travel @ $v = 0.8\,c$

a) $\gamma = \frac{5}{3} = 1.667$

b) proper half-life $= 1.8 \times 10^{-8}\,s = t_{1/2\,proper} = t'_{1/2}$

$t_{1/2\,lab} = t_{1/2} = \gamma\, t'_{1/2} = 3 \times 10^{-8}\,s$

c) 36 m in lab frame : $\Delta t = \frac{36\,m}{(0.8\,c)} = 15 \times 10^{-8}\,s = 5 \times t_{1/2}$

Number left $= 32{,}000 \left(\frac{1}{2}\right)^5 = 1000$

in the absence of time dilation $t_{1/2} = t'_{1/2} = 1.8 \times 10^{-8}\,s$

$\therefore$ # of half-lives $= 25/3 \cong 8.33$

So; Number left $= 32{,}000 \left(\frac{1}{2}\right)^{25/3} \simeq 100$

10. event 1 : $(x_1, y_1, z_1) = (0,0,0)$ $\quad (x'_1, y'_1, z'_1) = (0,0,0)$

event 2 : $(x_2, y_2, z_2) = (100, 0, 0)\,m$ $\quad (x'_2, y'_2, z'_2) = (125, 0, 0)\,m$

$\Delta t = 0$ $\quad \Delta t' = ? = \gamma\left(\Delta t - \frac{u\Delta x}{c^2}\right)$

$\Delta x = 100\,m$ $\quad \Delta x' = 125\,m$

$\Delta x' = \gamma(\Delta x - u\cancel{\Delta t}^{\,0}) = \gamma \Delta x$ Solve for $u \Rightarrow u = \frac{3}{5}c$

$\Delta t' = \gamma\left(\cancel{\Delta t}^{\,0} - \frac{u\Delta x}{c^2}\right) = -2.5 \times 10^{-7}\,s$ $\therefore$ NOT simultaneous