PH 1130

STUDY GUIDE #3

Relativistic Mechanics

I. Mass

Study Section 3.2

You may have heard that the mass of an object varies with its speed. That is an old fashioned point of view. The word "mass" refers to the rest mass of an object. It is a scalar quantity and is independent of observer or speed.

II. Momentum

A. Study Section 3.3

Notice on the last half of page 47, that T&Z are analyzing what would be the case if special relativity were NOT the law of nature. On the next page they describe the actual physics.

Forget the final paragraph. It is unnecessary and confusing.

B. When Newton wrote the second law, he wrote $\mathbf{F} = d\mathbf{p}/dt$ rather than $\mathbf{F} = m\mathbf{a}$. Relativistically, he was correct, if momentum is correctly defined. We also insist that a useful definition preserve the law of momentum conservation. T&Z takes you through a fairly detailed analysis of a particular type of collision to demonstrate that a plausible definition of momentum is $\mathbf{p} = \gamma m\mathbf{v}$. At slow speeds, $\gamma = 1$, and the newtonian (classical) definition emerges.

III. Energy

Study Sections 3.4 through 3.8

A. A body at rest (in a given Reference Frame) has energy $E = mc^2$, the equation that **everyone** knows. If the body is moving, the total energy is $E = \gamma mc^2$. The kinetic energy is the difference between the total energy and the rest-mass energy. At slow speeds, that is just the old familiar $\frac{1}{2}mv^2$.
(The binomial expansion of γ is used to arrive at the low-speed approximation in equation 3.11)

B. In example 3.3, T&Z are at pains to solve this problem nonrelativistically first, and then to solve it correctly. This approach can be helpful because; a) it gives you a chance to review the solution of an non-relativisitic elastic collision problem, which you studied in PH 1110 or PH 1120; b) you can see the pronounced difference in the results of a relativisitc analysis. Nonetheless, this dual approach makes for a very long solution, it assumes that you are reasonably familiar with the nonrelativistic solution. It may be hard to see the forest through the trees. Just be aware that they solve the example **twice**, incorrectly the first time and <u>correctly</u> the second. Don't get lost! Whatever, you do, **don't** memorize any of the equations on page 54; they're all very special cases, and, although correct, they are so complicated and ugly that it's hard to learn from or to use.

C. It is "better" to write equation 3.23 in the form

$E^2 - (pc)^2 = (mc^2)^2$

The right-hand-side is a scalar, and an invariant quantity; mc^2 is the same number in every <u>inertial</u> reference frame. Although one observer may measure a different energy than another, and a different momentum, this equation tells us that the particular combination on the 1eft-hand-side of the equation is the same number to everyone. Thus, momentum and energy are intimately related in much the same way that space and time are.

$E^2 - (pc)^2 = (mc^2)^2$, an invariant;

$(c\Delta t)^2 - (\Delta s)^2 = (\Delta\tau)^2$, an invariant.

The value of the invariant space-time interval depends on what the separation of events in space and time is; the value of the invariant energy-momentum quantity is always the mass of the particle.

Mathematically, t, x, y, z, are the components of a "four-vector" which is the four-dimensional analog to the ordinary three-dimensional vectors you're accustomed. Now, E, P_x, P_y, P_z, are also the components of a four-vector, so it's not too surprising that their magnitudes, the invariant interval or the mass, respectively behave in the same way.

D. The name of the unit "electron-volt" tells you how much energy an eV is: multiply the charge of an electron by one volt:

$1 \text{ eV} = 1.6 \times 10^{-19}$ coulombs $\times$ 1 volt $= 1.6 \times 10^{-19}$ J.

E. Masses of particles are invariably expressed in electron-volts. When someone says, "The mass of a proton is 938.3 MeV", she means that the rest-mass energy of a proton, mc^2, is 938.3 MeV.

F. The title of Section 3.6 is unfortunate. Mass is not "converted" to anything; mass is the energy of system at rest. The total energy of an isolated system is conserved. If something happens to the system, you set the total energy of the system **before** equal to the total energy **after**. If the system is not isolated, but external work is performed on it, the total initial energy plus the net work performed is equal to the total final energy. That's the entire content of Section 3.6. The examples are useful illustrations, (which you should be sure you understand), but there is no other physics than energy conservation.

It is true that processes occur that conserve momentum and energy, in which the *rest-mass* of the system before the process is different from the *rest mass* after the process. In the relativistic mechanics, the nonconservation of mass is logically necessary. Indeed, even a mass at rest has an energy equal to mc^2. This rest energy can be fully or partially converted into some other, more familiar forms of energy, such as kinetic energy. Nuclear processes of *Fission* and *Fusion* should be mentioned in this regard.

In *Fission* a heavy radioactive nucleus decays into several lighter nuclei with large kinetic energies and with the sum of *rest masses* of the decay products being smaller than the *rest mass* of the initial nucleus (Example 3.6). This is the case for heavy elements in the Periodic Table. A large-scale self-sustainable fission is called a *chain reaction* and we have either a nuclear reactor or an atomic bomb.
For light nuclei and in chemistry we frequently see the opposite situation: two lighter particles can unite to form a stable heavier particle. We have to do work to pull this particle apart. This work is called the *binding energy*. The *rest mass* of the composite particle in this case is less than the sum of rest masses of two initial particles. Example 3.7 discusses binding energy in chemistry. At the nuclear level, we can collide light nuclear particles with enough energy to overcome their initial repulsion and *fuse* them together. This *Fusion process* is used in a hydrogen bomb and may become a source of power for humanity. Chapter 13 (see Fig. 13.12) contains more details.

G. *Study Section 3.7.* Of course the concept of the relativistic force **F** is not less important than a concept of relativistic momentum **p.** Using the definition **F** = d**p**/dt, simplifies the transition to relativity in a sense that this definition is analogous to nonrelativistic one and that the work-energy theorem is being easily carried over to relativity. Study Example 3.9 carefully.

H. *Section 3.8* considers the limiting case of massless particles. Yes, they are perfectly legitimate in relativity! The simplest example is light particles or photons. We'll deal with photons in the next part of the course (Ch5).