PH1130 **In-class problems** **9/6/07**

1. A 60 kg person is standing at rest on level ground. How fast would she have to run to

a)double her total energy?

b)increase her total energy by a factor of 10?

2. A relativistic football game is being played in a universe where the speed of light is c = 20 m/s. A player on the field throws a football with rest mass 0.4 kg downfield, at a speed v = 0.8 c. Soon after the ball is thrown, it explodes into two equal parts, one of which moves in the same direction that the original football was travelling, and the other moving in the opposite direction (back toward the ball thrower). In the reference frame of the original football, these two parts have the same speed 0.3 c.

a) Determine the momentum of the football before the explosion (in MKS units).

b) Determine the kinetic energy of the football before the explosion (in MKS units).

c) Determine the total rest mass of the two parts after the ball explodes.

d) Determine the velocity of the part that goes back toward the thrower.

e) Determine the velocity of the part that goes in the same direction that the original football was travelling.

f) Determine the total momentum of the two parts after the explosion.

g) How do the answers to parts a and f compare? Explain.

PH1130 **Extra in-class problems** **9/6/07**

1. a) $E_0 = m_0 c^2$

$$2E = mc^2 = 2m_0 c^2$$

$$\therefore m = 2m_0 \rightarrow \frac{m_0}{\sqrt{1 - v^2 / c^2}} = 2m_0$$

$$\frac{1}{4} = 1 - \frac{v^2}{c^2} \rightarrow \frac{v^2}{c^2} = \frac{3}{4} \rightarrow v = c\sqrt{3/4}$$

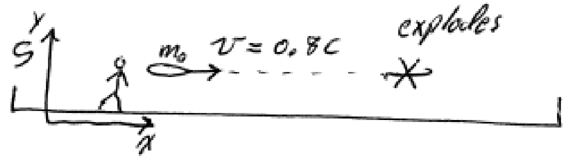
$$v = 0.866\ c = 2.60 \times 10^8 \text{ m/s}$$

b) $10\ m_0 c^2 = mc^2 = \dfrac{m_0}{\sqrt{1 - v^2 / c^2}} c^2$

$$1 - \frac{v^2}{c^2} = \frac{1}{100} \rightarrow \frac{v^2}{c^2} = \frac{99}{100}$$

$$v = c\sqrt{\frac{99}{100}} = 0.995\ c = 2.98 \times 10^8 \text{ m/s}$$

2.

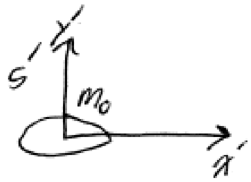


$m_0 = 0.4$ kg

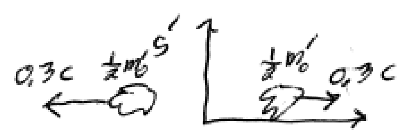
$c = 20$ m/s

$\gamma = \frac{1}{\sqrt{1-(.8)^2}} = 1.667$

In football's frame S':



before



after

a) $P_x = \gamma m_0 v = (1.667)(0.4)(0.8)(20) = \boxed{10.67 \ \frac{kg \cdot m}{s}}$

b) $E_K = (\gamma - 1)E_0 = (1.667-1)(0.4)(20)^2 = \boxed{106.7 \text{ J}}$

c) Use cons. energy in S' frame:

$$m_0 c^2 = \gamma'\left(\frac{m_0'}{2}\right)c^2 + \gamma'\left(\frac{m_0'}{2}\right)c^2$$

where $\gamma' \equiv \frac{1}{\sqrt{1-(.3)^2}} = 1.048$

$\therefore \ m_0 = \gamma' m_0'$

$$m_0' = \frac{1}{\gamma'} m_0 = \frac{0.4}{1.048} = \boxed{0.3817 \text{ kg}}$$

The decreased rest mass has been converted into kinetic energy.

d) Labeling the left part 1,

$$u_{1x} = \frac{u'_{1x} + v}{1 + \frac{u'_{1x} v}{c^2}} = \frac{(-0.3 + 0.8)c}{1 - (0.3)(0.8)} = \boxed{0.658\,c}$$

e) Labeling the right part 2,

$$u_{2x} = \frac{u'_{2x} + v}{1 + \frac{u'_{2x} v}{c^2}} = \frac{(0.3 + 0.8)c}{1 + (0.3)(0.8)} = \boxed{0.887\,c}$$

f) $$p_x^{after} = p_{1x} + p_{2x} = \gamma_1 \left(\frac{m'_0}{2}\right) u_{1x} + \gamma_2 \left(\frac{m'_0}{2}\right) u_{2x}$$

where $$\gamma_1 \equiv \frac{1}{\sqrt{1 - \frac{u_{1x}^2}{c^2}}} = 1.328 \qquad \gamma_2 \equiv \frac{1}{\sqrt{1 - \frac{u_{2x}^2}{c^2}}} = 2.165$$

$$p_x^{after} = \left(\frac{0.3817}{2}\right)\left[(1.328)(0.658) + (2.165)(0.887)\right](20)$$

$$p_x^{after} = 10.67 \ \frac{kg \cdot m}{s}$$

g) Comparing parts (a) and (f), we see

$$p_x^{before} = p_x^{after}$$

which is what we expect from conservation of momentum.