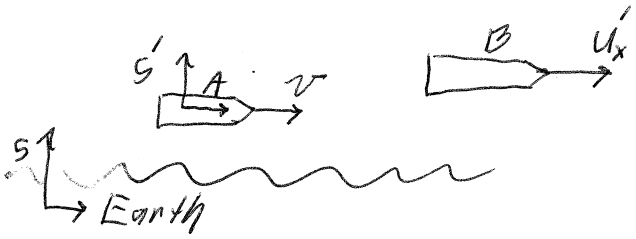
PH1130 Practice Problem #1

Spaceship A is travelling by the earth in an easterly direction at a speed of 0.8 c relative to the earth.

a) If spaceship B passes A at a speed of 0.5 c east relative to A, what is B's speed relative to the earth?



$v = 0.8\ c$

$u_x' = 0.5\ c$

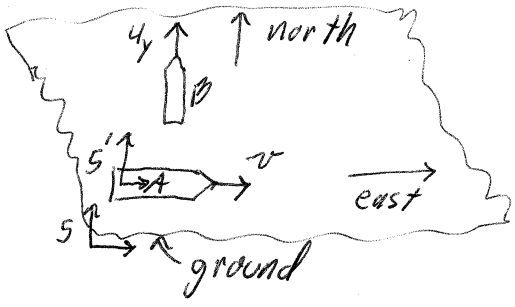
Need u_x

$\gamma = 1.667$

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{(0.5 + 0.8)\,c}{1 + (0.5)(0.8)} = \boxed{0.928\ c}$$

b) Suppose that B was instead moving north at 0.5 c relative to the earth. What would B's speed then be relative to A?

top view



In the earth frame (S) A is moving due east and B is moving due north.

$\therefore$ $u_x = 0$, $u_y = 0.5\ c$ } motion of B in S frame

Now find motion of B in S' frame:

$$u_{x'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = -v = -0.8\ c$$

$$u_{y'} = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} = \frac{u_y}{\gamma} = \frac{0.5\ c}{1.667} = 0.3\ c$$

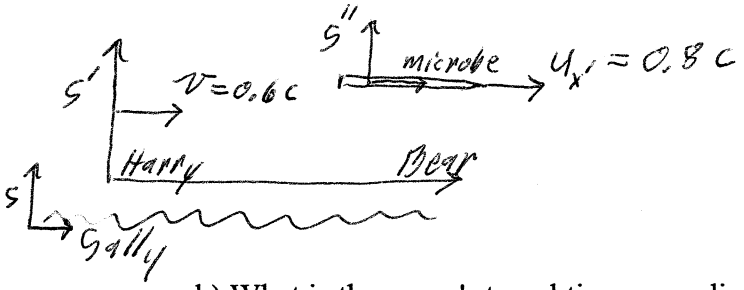
The speed of B in S' is then

$$u' = \sqrt{u_{x'}^2 + u_{y'}^2} = \sqrt{(0.8)^2 + (0.3)^2}\ c = \boxed{0.854\ c}$$

PH1130 Practice Problem 2

Harry is running at 0.6 c relative to the ground and is chasing a bear who is also running at 0.6 c in the same direction. He determines that the bear is 120 meters in front of him. He shoots an arrow at a speed of 0.8 c relative to himself directly at the bear. There is an intelligent microbe on the arrow, and Sally is hiding in the bushes, observing all the action. Answer the following:

a) What is the arrow's speed according to Harry (H), Sally (S), and the microbe (M)?



H; $u_{x'} = 0.8\,c$

S: $u_x = \dfrac{(0.8+0.6)c}{1+(0.8)(0.6)} = 0.946\,c$

M; $u_{x''} = 0$

b) What is the arrow's travel time according to each of the observers in (a)?

event		x	t	x'	t'	x''	t''
#1	arrow shot	0	0	0	0	0	0
#2	arrow hits bear		□	120 m	$\dfrac{120}{(0.8)(3\cdot10^8)}$	0	□

For motion of S' relative to S: $\gamma = \dfrac{1}{\sqrt{1-(0.6)^2}} = 1.25$

For motion of S'' relative to S': $\gamma' = \dfrac{1}{\sqrt{1-(0.8)^2}} = 1.667$

Harry: $t_2' = \dfrac{120\ m}{(0.8)(3\cdot10^8\ m/s)} = \boxed{5\cdot10^{-7}\ s}$

Sally: $t_2 = \gamma\left(t_2' + \dfrac{v x_2'}{c^2}\right) = (1.25)\left[5\cdot10^{-7} + (0.6)\dfrac{120}{3\cdot10^8}\right] = \boxed{9.25\cdot10^{-7}\ s}$

Microbe: t_2'' is proper time, so $t_2' = \gamma' t_2''$

$\therefore\ t_2'' = \dfrac{t_2'}{\gamma'} = \dfrac{5\cdot10^{-7}\ s}{1.667} = \boxed{3\cdot10^{-7}\ s}$

c) What is the separation between Harry and the bear according to each of the observers in (a)?

H: $L' = \boxed{120\ m}$

S: $L = 120\sqrt{1-(0.6)^2} = \boxed{96\ m}$

M: $L'' = 120\sqrt{1-(0.8)^2} = \boxed{72\ m}$

d) If the arrow is 0.9 meters long in its rest frame, what is its length according to each of the three observers?

H: $\ell' = (0.9)\sqrt{1-(0.8)^2} = \boxed{0.54\ m}$

S: $\ell = (0.9)\sqrt{1-(0.946)^2} = \boxed{0.29\ m}$

M: $\ell'' = \boxed{0.9\ m}$