Prob. 1 Calculate the net kinetic energy of the decay products released in the beta decay of ^{14}C,

$$^{14}C \rightarrow {}^{14}N + e + \nu$$

take the atomic mass of ^{14}C to be 14.00324 u, and that of ^{14}N to be 14.00307 u

$$\frac{Q}{c^2} = M_{^{14}C} - M_{^{14}N} = 1.7 \cdot 10^{-4}\ u$$

$$Q \approx \left(931.5\ \frac{MeV}{u}\right)(1.7 \cdot 10^{-4}\ u) \approx \boxed{0.158\ MeV}$$

Prob. 2 Calculate the binding energy per nucleon of ^{40}Ca (Z = 20). Take the atomic mass of ^{40}Ca to be 39.96259 u

$$\frac{BE}{c^2} = Z M_H + N m_n - M_{^{40}Ca}$$

$$= (20)(1.007825) + (20)(1.008665) - 39.96259\ u$$

$$\frac{BE}{c^2} = 0.367\ u$$

$$BE = \left(931.5\ \frac{MeV}{u}\right)(0.367\ u) = 342\ MeV$$

$$\frac{BE}{\text{nucleon}} = \boxed{8.55\ MeV}$$

Prob. 3 ^{40}K decays into ^{40}Ar by positron decay (β^+) with a half-life of 1.28 billion years.

During early times on the earth, some ^{40}K was trapped in some molton lava from a volcano, and this solidified into rock which originally contained no gas. Researchers broke open the rocks, and found trace amounts of the gas ^{40}Ar. If the ratio of potassium (K) to argon (Ar) is 1/3 (by number of atoms), how old is the rock?

$$N_{Ar} + N_K = N_{Tot} = N_K(0)$$

$$\frac{N_{Ar}}{N_K} = 3$$

$$\therefore \quad 3N_K + N_K = N_K(0)$$

$$4N_K = N_K(0)$$

$$N_K = \frac{1}{4} N_K(0)$$

This is two half-lives $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$$\text{age} = 2\,T_{1/2} = \boxed{2.56 \text{ billion yrs.}}$$

Or,

$$\frac{1}{4} = e^{-\lambda t}$$

$$4 = e^{\lambda t}$$

$$\ln 4 = \lambda t$$

$$t = \frac{\ln 4}{\lambda} \qquad \text{but} \quad \lambda = \frac{\ln 2}{T_{1/2}}$$

so $t = \frac{\ln 4}{\ln 2} T_{1/2} = \frac{2 \ln 2}{\ln 2} T_{1/2} = 2\,T_{1/2}$

as before