Ohm's law is our handy tool for analyzing simple resistor circuits

$$V = IR$$

V is the voltage applied to the resistor, I is the current through the resistor, and R is the resistance of the resistor. We also say that V is the "voltage drop" across the resistor.

If two resistors are connected in series to a battery then the resistors carry the same current but each has its own voltage drop, and the sum of the voltage drops equals the battery voltage

$$V_1 = IR_1 \,, \qquad V_2 = IR_2 \,, \qquad V = V_1 + V_2 = I(R_1 + R_2)$$

from which we obtain the **equivalent resistance of series resistors**

$$R_{eq} = R_1 + R_2 \,, \qquad V = IR_{eq} \,, \qquad I = V/R_{eq}$$

For a given V, finding the equivalent resistance enables us to determine the current.

A real battery has a small internal resistance, and is modeled as an ideal battery $\mathcal{E}$ in series with a resistor r. When attached to a circuit, the current I corresponds to a small internal voltage drop $v = Ir$.

If you attach a real battery to a resistor, the two resistors r and R are in series

$$v = Ir \,, \qquad V = IR \,, \qquad \mathcal{E} = v + V \,, \qquad V = \mathcal{E} - Ir \,, \qquad R_{eq} = r + R$$

The small voltage drop across the internal resistor lowers the apparent battery voltage, which is the voltage V that appears at the terminals of the battery.

Usually $r \ll R$ and its effect on the circuit current and voltage is negligible. As an example, suppose

$$r = 0.1\Omega \,, \qquad R_{cir} = 2k\Omega \,, \qquad \mathcal{E} = 1.5V$$

The equivalent resistance differs from the circuit resistance by 1 part in 20,000

$$R_{eq} = r + R_{cir} = 2000.1\ \Omega$$

The circuit current is slightly altered by the presence of r. Compare cases with and without r

$$I = \mathcal{E}/R_{cir} = 0.75\ mA$$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_{cir}(1 + r/R_{cir})} \cong \frac{\mathcal{E}}{R_{cir}}\left(1 - \frac{r}{R_{cir}}\right) = 0.74996\ mA$$

The terminal voltage also is slightly lowered

$$V = \mathcal{E} - Ir = 1.49993\ V$$

When would the battery internal resistance matter? Answer: when the circuit resistance is also small, comparable to the internal battery resistance

$$if\quad R \approx r \;\; then \;\; v \approx V \;\; and \;\; V = \mathcal{E} - v \;\; is\; much\; less\; than\; \mathcal{E}$$

in which case the terminal voltage and the circuit current are significantly altered.

Parallel resistors have the same voltage drop but different currents

$$V = I_1 R_1\,, \qquad V = I_2 R_2$$

The total current is the sum of the currents

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \qquad \frac{V}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = R_{eq}\,, \qquad R_{eq} < R_1\,, \qquad R_{eq} < R_2$$

The equivalent resistance is less than either of the two resistors! This makes sense: parallel resistors offer more avenues for the current to flow. For a given V, more current implies less resistance.

Alternate ways of writing R_{eq} are

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}\,, \qquad or \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For a given V, finding the equivalent resistance enables us to determine the current

$$I = V/R_{eq}$$

An ammeter measures the amount of current in a circuit. You must "cut" the circuit and insert the ammeter so that the circuit current flows through the ammeter. An ideal ammeter's internal resistance is zero, so that it has no effect on the circuit. Real ammeters have a small internal resistance that can be neglected for most applications. The ammeter is connected in series with the circuit, and there will be a small voltage drop across the ammeter's resistance. <u>For a given source voltage</u> V

$$V = V_{cir} + V_{am} = IR_{cir} + IR_{am} = I(R_{cir} + R_{am}), \qquad V_{cir} = V - V_{am}$$

the ammeter increases the "load" resistance (the resistance of the circuit plus ammeter), which decreases the current I, which in turn reduces the voltage drop across the circuit.

A voltmeter measures the voltage difference between two points in a circuit. Unlike the ammeter, we simply touch the voltmeter's probes to the points. The voltmeter senses the voltage difference between the two points. An ideal voltmeter has infinite internal resistance, so that it has no effect on the circuit. A real voltmeter has a large resistance, typically $r > 10M\Omega$. The voltmeter is connected in parallel with a circuit. A small current flows through the voltmeter. <u>For a given source current</u> I

$$I = I_{cir} + I_{vm} = \frac{V}{R_{cir}} + \frac{V}{R_{vm}} = V\left(\frac{1}{R_{cir}} + \frac{1}{R_{vm}}\right), \qquad I_{cir} = I - I_{vm}$$

the voltmeter decreases the load resistance, causing a slight decrease in the circuit current.

Circuit Power. Consider a battery of voltage V delivering current I to a resistor. From the definition of current we get the amount of charge dQ transferred in time dt. This charge travels across a voltage drop and experiences a loss in potential energy dU given by $dU = VdQ$

$$I = dQ/dt \;, \qquad dQ = I\,dt \;, \qquad dU = VdQ = VI\,dt$$

Where does the energy go? The energy is lost in the circuit as heat. The rate at which the energy is transferred is the power P

$$P = \frac{dU}{dt} = VI \;, \qquad units\ of\ V \cdot A = \frac{J}{C}\frac{C}{s} = \frac{J}{s} = W, \qquad Watts$$

As an example, a battery delivers energy to a resistor at the rate P and heats the resistor

$$V = IR \;, \qquad P = VI = I^2R = V^2/R$$

If the resistor is a light bulb, the filament becomes hot, its atoms become very agitated and transition to excited states. The atoms quickly return to their ground states by emitting photons of light. Some are infrared photons (invisible), others are visible photons, and still others are ultra-violet photons.

The process of atomic excitation and emission repeats as long as energy is supplied by the battery. Some of the energy goes into heating the filament until it begins to glow. When the rate at which energy is delivered equals the rate at which energy is radiated, the bulb assumes a steady brightness.

The power delivered to series resistors depends on their individual voltage drops

$$V_1 = IR_1 \;, \qquad V_2 = IR_2 \;, \qquad P_1 = V_1I = I^2R_1 \;, \qquad P_2 = V_2I = I^2R_2, \qquad P = I^2R_{eq}$$

The power delivered to parallel resistors depends on their individual currents

$$I_1 = \frac{V}{R_1} \;, \qquad I_2 = \frac{V}{R_2} \;, \qquad P_1 = I_1V = \frac{V^2}{R_1} \;, \qquad P_2 = I_2V = \frac{V^2}{R_2}, \qquad P = \frac{V^2}{R_{eq}}$$

The power delivered to a circuit can also be expressed in terms of the circuit's equivalent resistance

$$P = I^2R_{eq} = V^2/R_{eq}$$

One circuit can output power to another circuit. A battery is a circuit. A generator is a circuit. A resistor network is a circuit. If you hook two circuits together with terminal voltage V, the one that outputs current I outputs power $P = IV$. The other circuit inputs the power, absorbs it.

A generator can charge a battery; it pours energy into the battery at the rate $P = IV$. Similarly one capacitor can charge another capacitor at the rate $P = I(V_1 - V_2)$.

You can divide a circuit into two parts and ask which part outputs power to the other part. For example a battery's ideal source $\mathcal{E}$ delivers power to its internal resistance and to an external resistor

$$P_r = I^2r \;, \qquad P_R = I^2R$$