If you move a bar magnet toward a loop of wire, it causes an electric current to flow in the wire! This physical process is the basis of electric generators, and is described by Faraday's law. Faraday's law is one of the fundamental laws of electromagnetism. The law involves the rate of change of "magnetic flux". To understand the law we must introduce the concept of flux and magnetic flux.

Imagine light flowing through a window. We define the "flux" of light flowing through the window as the product of the intensity of the light multiplied by the area of the window

$$flux = intensity \times area$$

Imagine a uniform magnetic field of "intensity" B whose field lines "flow" through a window of area A. Suppose the field lines are perpendicular to the plane of the window. The magnetic flux Φ through the window is defined to be

$$\Phi = BA\,, \qquad units\ of\ [T \cdot m^2]$$

where A is the area of the window of width W and length L

$$A = WL$$

Now imagine a second window oriented at an angle θ with the first window and placed so that the "rectangular tube" of field lines filling the first window also fills the second window. The second window has the same width W but greater length L'. The two lengths are related by

$$L = L' \cos\theta$$

and by substitution, we see that the two areas are related by

$$A = WL = WL' \cos\theta = A' \cos\theta\ , \qquad where\ A' = WL'$$

Since the flux is the same through both windows, we may write the flux through the second window as

$$\Phi = BA = BA' \cos\theta$$

The factor $\cos\theta$ converts the area of the tilted window into the area of the perpendicular window.

In textbooks we find the formula for magnetic flux written as

$$\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where θ is the angle between the direction of the field and the direction in which the window is facing. The vector $\vec{A}$ is a vector area, an area A whose direction is specified by a unit vector perpendicular ("normal") to the plane of the area

$$\vec{A} = A\,\hat{u}_{\perp}$$

Having defined magnetic flux, we now consider a change in flux $\Delta\Phi$ occuring in a time Δt. The average rate of change of flux over the interval of time becomes the instantaneous rate of change when we let Δt become infinitesimal

$$\lim_{\Delta t \to 0} \frac{\Delta\Phi}{\Delta t} = \frac{d\Phi}{dt}$$

What might cause a change in flux? If we look at the formula for flux

$$\Phi = BA\cos\theta$$

there are three variables B, A and θ. If one or more of these changes, the flux will change; the field strength B might change, the area of the window A might change and the angle θ might change.

Considered separately, the changes in flux are

$$d\Phi = dBA\cos\theta\,, \qquad d\Phi = BdA\cos\theta\,, \qquad d\Phi = BA(-\sin\theta)d\theta$$

If all three variables are changing simultaneously, the total change in flux is

$$d\Phi = dBA\cos\theta + BdA\cos\theta - BA\sin\theta\, d\theta$$

and if the changes occur in a time dt then the total rate of change of flux is

$$\frac{d\Phi}{dt} = \frac{dB}{dt}A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt}$$

We are now ready to state Faraday's law: the rate of change of magnetic flux through a closed loop (the loop is the "window") induces an electromotive force $\mathcal{E}$ (a voltage) around the loop given by

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

What does this mean? If you move a bar magnet toward a loop of wire, it increases the flux through the loop, which induces a voltage around the loop, which causes an electric current to flow in the wire!

It means you no longer need to comb your cat on a dry day to create electric current, and you no longer need to buy a battery to create electric current.

All you need is a bar magnet and loop of wire and you can power your droid, your light bulb, your toaster, your heater and your hybrid vehicle. In other words, you have the technology to build a generator and generate electricity.

Example: a uniform magnetic field increases in strength at the rate

$$\frac{dB}{dt} = 0.03\frac{T}{s}$$

The field lines are perpendicular to a loop of wire of area $A = 0.01\ m^2$. The induced emf is

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{dB}{dt}A\cos\theta = -\left(0.03\frac{T}{s}\right)(0.01m^2)\cos(0°) = -3.0 \times 10^{-4}V$$

How do we interpret the minus sign? Using your <u>left hand</u> ruler, point your thumb in the direction of $\Delta\vec{B}$; the fingers of your left hand curl in the direction of the induced current.

Watch out for the gotcha: you must point your left thumb in the direction of $\Delta\vec{B}$, not $\vec{B}$!

The changing flux "inserts" an imaginary battery of voltage $\mathcal{E}$ into the circuit. If the loop is a circle of radius r, the emf creates a uniform electric field along the loop of magnitude

$$E = \frac{V}{d} = \frac{\mathcal{E}}{2\pi r}$$

If the resistance of the loop of wire is $R = 0.02\ \Omega$, the induced current is (by Ohm's law)

$$I = \frac{\mathcal{E}}{R} = -1.5 \times 10^{-2}A$$

Example: the North face of a magnet passes over a loop of wire so that the portion of the loop's area exposed to the magnet's field increases at the rate

$$\frac{dA}{dt} = 1.0 \times 10^{-3}\frac{m^2}{s}$$

The field strength is $B = 0.5T$ and the face of the magnet is parallel to the plane of the loop. The induced emf is

$$\mathcal{E} = -\frac{d\Phi}{dt} = -B\frac{dA}{dt}\cos\theta = -(0.5T)\left(1.0 \times 10^{-3}\frac{m^2}{s}\right)\cos(0°) = -0.5 \times 10^{-3}V$$

Which way does current flow around the loop? Point your left thumb in the direction of $\Delta\vec{A}$; the fingers of your left hand curl in the direction of the induced current.

Watch out for the gotcha: you must point your left thumb in the direction of $\Delta\vec{A}$, not $\vec{A}$!

In the above examples we took the field $\vec{B}$ and the vector area $\vec{A}$ to be pointing in the same direction, so that the angle $\theta = 0$. This is the simplest and easiest case.

What if the field $\vec{B}$ points in the opposite direction to the vector area $\vec{A}$? Then

$$\cos\theta = \cos(180°) = -1$$

applies a factor of -1 to the result, which flips the direction of the induced current. For all cases, keep a simple picture in your head, and ask yourself: which way is the flow (the flux) increasing? Point your <u>left thumb</u> in that direction and your fingers will curl in the direction of the current.