A charged particle of velocity $\vec{v} \perp \vec{B}$ moves in a circle in a uniform magnetic field

$$F = qvB = \frac{mv^2}{r} = m\omega^2 r$$

The radius of the circle is

$$r = \frac{mv}{|q|B}$$

The angular speed ω is known as the cyclotron frequency

$$\frac{v}{r} = \omega = \frac{|q|B}{m}$$

after the cyclotron, a particle accelerator invented by Lawrence in 1932.

If the velocity $\vec{v}$ has a component parallel to the magnetic field the particle will move in a helix, a spiral

$$\vec{B} = B\hat{z}\,, \qquad \vec{v} = \vec{v}_{\perp} + \vec{v}_{\parallel}$$

$$\vec{F} = q\vec{v} \times \vec{B} = qv_{\perp}B(-\hat{r})$$

The parallel component of the velocity does not contribute to the force, but does give the particle a constant drift speed in a direction parallel to the magnetic field.

We showed in the previous lecture how Thompson applied the above formula to infer that cathode rays are speedy electrons, and he measured the ratio of charge to mass of the electron

$$qvB = \frac{mv^2}{r}, \qquad \rightarrow \frac{e}{m} = \frac{q}{m} = \frac{v}{rB}$$

By creating a uniform field B, then accelerating the (as yet unidentified) electrons through a known potential V, the electron speed could be related by the work-energy theorem $\Delta U = \Delta K$ to the potential

$$eV = \frac{mv^2}{2}\,, \qquad v^2 = \frac{e}{m}2V\,, \qquad \rightarrow \left(\frac{e}{m}\right)^2 = \left(\frac{1}{rB}\right)^2 v^2 = \left(\frac{1}{rB}\right)^2 \frac{e}{m}2V\,, \qquad \rightarrow \frac{e}{m} = \left(\frac{1}{rB}\right)^2 2V$$

Thompson measured the deflection of the electron beam (the radius r), and computed e/m.

A velocity selector subjects a stream of charged particles to uniform electric and magnetic fields that are oriented in such a way that the forces exerted by the two fields on a charged particle cancel

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right), \qquad F_E = qE\,, \qquad F_B = qvB\,,$$

$$F_E - F_B = 0\,, \qquad qE - qvB = 0\,, \qquad v = \frac{E}{B}$$

for a particular particle speed v. If the beam consists of particles of various speeds, only those particles having $v = E/B$ will move along a straight line and pass through the selector; other particles will be deflected away. The resulting beam will then consist of particles moving at the same speed v.

For example, for positive charge q, velocity in the $+x$ direction , electric field in the $-y$ direction, the magnetic field must point in the $-z$ direction

$$\vec{v} = v\hat{x}, \qquad \vec{E} = -E\hat{y}\,, \qquad \vec{B} = -B\hat{z}$$

Note: sometimes you will see the unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ written as $\hat{x}, \hat{y}, \hat{z}$, names that are somewhat more descriptive. Do not let it confuse you, they are still the same old familiar unit vectors!

Working with a velocity selector, Thompson obtained the ratio of electron charge to mass

$$v^2 = \frac{2eV}{m} = \left(\frac{E}{B}\right)^2\,, \qquad \frac{e}{m} = \frac{1}{2V}\left(\frac{E}{B}\right)^2$$

A mass spectrometer accepts a beam of particles having the same speed and separates them by mass by passing the beam through a region of uniform magnetic field B'.

In this region particles of mass m move along semi-circular arcs or radius

$$r = \frac{mv}{qB'} = \frac{m}{qB'}\frac{E}{B}\,, \qquad \leftarrow \quad v = \frac{E}{B}$$

Particles of greater mass move along arcs of greater radius. The particles are intercepted by a particle detector. A range of masses m_1 to m_2 is distributed along a particle detector at distances ranging from $2r_1$ to $2r_2$ from the beam entrance, forming a "spectrum"

$$r_1 = \frac{m_1}{qB'}\frac{E}{B}\,, \qquad r_2 = \frac{m_2}{qB'}\frac{E}{B}$$

A magnetic field exerts a force on a current carrying wire. Consider a uniform magnetic field $\vec{B}$ pointing into the page, a straight wire laying in the plane of the page, having cross section A and carrying current I consisting of particles of charge q moving with drift speed v_d along the wire. The force acting on a charge is perpendicular to both the wire and the field

$$F = qv_dB$$

The total free charge Q in a segment of wire of length L is

$$Q = qN = qnAL$$

where n is the number density of free charges.

The total force acting on the segment of wire (for angle $\theta = 90°$ between velocity and field)

$$F = Qv_dB = (qnAL)v_dB = ILB\,, \qquad \leftarrow \quad I = JA = nqv_dA$$

The more general formula

$$\vec{F} = I\vec{L} \times \vec{B}\,, \qquad d\vec{F} = I\,d\vec{L} \times \vec{B}\,, \qquad \vec{F} = \int d\vec{F}$$

applies to any angle θ between the velocity and field vectors.

For curvilinear wires the differential form $d\vec{F}$ may be used to sum the force over a length of wire. It may also be used to approximate the field created by a short segment $\Delta\vec{L}$ of a curved wire

$$\Delta\vec{F} \cong I\,\Delta\vec{L} \times \vec{B}$$

A rectangular current loop that rotates about an axis experiences no net force when placed in a uniform magnetic field, but does experience a torque. This loop forms the "rotor" of a simple electric motor. In Y&F figure 27.31 page 936, forces on opposite sides of the loop are equal and opposite so that the sum of forces is zero.

When the loop faces a direction at an angle φ from the direction of the magnetic field, the torque is

$$\tau = 2F\frac{b}{2}\sin\varphi\ , \qquad F = IaB$$

At $\theta = 0$ the torque is zero, and at $\varphi = \pi/2$ the torque is at a maximum.

The loop will experience an angular acceleration, where M is the moment of inertia of the loop

$$\alpha = \frac{\tau}{M}$$

which will cause the loop to rotate.

When the loop passes through the $\varphi = 0$ orientation, the angular acceleration will reverse direction (from clockwise to counter-clockwise, etc), and the loop will experience an angular oscillation.

However, if a mechanism is provided to flip the magnetic field as the loop passes through $\varphi = 0$, the acceleration will always act in the same direction (in this case, clockwise).

In this case the loop will accelerate up to a maximum angular speed determined when external forces (such as friction or workload forces) compete with the torque to produce equilibrium.