Parallel Plate Capacitors and Capacitance. Parallel plates produce a uniform electric field.

We can charge two plates by attaching a battery of voltage V_{bat}. Positive charge $+Q$ accumulates on one plate while negative charge – Q accumulates on the other plate.

When fully charged, the voltage between the two plates equals the battery voltage $V = V_{bat}$.

The charge remains after the battery is disconnected from the plates. The plates "store" charge.

We call this ability to store charge "capacitance" C, and we call this device a "parallel plate capacitor".

The amount of charge is proportional to the applied battery voltage

$$Q = CV \quad or \quad C = \frac{Q}{V}\,, \qquad C\ has\ units\ of\ \frac{Coulomb}{Volt} = Farad,\ \mathcal{F} = \frac{\mathcal{C}}{\mathcal{V}}$$

The letter capital C does double duty as the Capacitance and as the unit of charge Coulomb.

The Electric Flux $\boldsymbol{\Phi}$ of a uniform electric field $\boldsymbol{E}$ passing perpendicularly through an imaginary flat surface of area A is defined as (sketch)

$$\Phi = EA$$

For a point charge Q and spherical surface enclosing the charge, the flux is found by noting that E is perpendicular to the surface at all points, therefore (sketch)

$$\Phi = EA = \frac{Q}{4\pi\epsilon_0 r^2}4\pi r^2 = \frac{Q}{\epsilon_0}$$

The flux is proportional to the charge. This is an example of Gauss' Law, which states that the total flux passing through a closed surface is proportional to the enclosed charge.

Applying the law to a capacitor of charge Q (that is $+Q$ on one plate and $-Q$ on the other plate), of plate area A and electric field strength E, the flux through a box enclosing the $+$ plate is (sketch)

$$\Phi = EA = \frac{Q}{\epsilon_0}$$

since the field is zero outside of the capacitor.

Formula for the capacitance. If the plates are separated by a distance d we have

$$V = Ed, \qquad or\ E = \frac{V}{d}$$

Inserting this expression for E into the preceding one

$$EA = \frac{V}{d}A = \frac{Q}{\epsilon_0} \quad or \quad Q = \left(\frac{\epsilon_0 A}{d}\right)V$$

Comparing this equation to $Q = CV$, we see that the capacitance must be

$$C = \frac{\epsilon_0 A}{d}$$

C is proportional to area and inversely proportional to the separation.

Size of a parallel plate capacitor. With $d = 1mm$, what area A stores $Q = 1C$ for $V = 1.5V$?

$$A = \frac{Qd}{\epsilon_0 V} = \frac{(0.001m)(1C)}{(8.85 \times 10^{-12}F/m)(1.5V)} = 75.3\ km^2\ , \qquad nearly\ the\ area\ of\ Worcester$$

Note how units cancel since $F = C/V$. See Wikipedia article on "Worcester, MA" for the area of Worcester $= 100\ km^2$.

Typical capacitors used in electronic circuits range in size from $1mF$ to $1pF$.

For the above example, a $3\mu F$ capacitor would have an area of

$$A = \frac{Qd}{\epsilon_0 V} = \frac{(0.001m)(3 \times 10^{-6}C)}{(8.85 \times 10^{-12}F/m)(1.5V)} = 216\ m^2$$

about the size of OH 107, the floor of this auditorium! That's too big for the circuit in your cell phone!

How do we make capacitors small?

1. You can use a smaller spacing d. However there is a limit to how small you can make the spacing (see Y&F section 24.4). A typical value is $d = 0.01\ mm$.

2. You can insert a material between the plates, a dielectric (an insulator), which becomes polarized and allows much more charge Q to be stored for a given voltage V, thus increases the capacitance. For example strontium titanate (see Y&F table 24.1) increases the capacitance by a factor of $K = 310$.

3. You can sandwich layers of metal foil and strontium titanate and roll them into a cylinder. Since the layers are incredibly thin, a hundred coils will have a diameter of a few millimeters.

Capacitors in Electric Circuits. Capacitors are used in electrical circuits, in large power distribution systems to micro-electronics. Their ability to store charge and to be rapidly charged and discharged makes them a versatile building block for a great variety of applications. In this course we barely scratch the surface. The condenser microphone (see Y&F figure 24.3) makes it possible for you to enjoy your favorite recording artists. Where do those thousands of songs stored on your iPods come from?

Let's study the behavior of simple capacitor circuits involving capacitors arranged in series and parallel.

Capacitors in series have the same current flowing "through" them. As a result, they acquire the same charge Q. In general they will not have the same voltage.

Capacitors in parallel see the same applied voltage, and will charge to the same voltage V. In general they will not have the same charge.

These facts, coupled with the additive property of voltages, enable us to determine all of the charges and voltages in simple capacitor circuits. The first step is to examine a given circuit and identify which capacitors see the same current (capacitors in series) and which capacitors see the same applied voltage (capacitors in parallel). Once you have determined this, you can proceed with your analysis.

Two capacitors C_1 and C_2 are connected in series with a battery V. The capacitors acquire the same unknown charge Q. We may write their unknown voltages as

$$V_1 = \frac{Q}{C_1}\,, \qquad V_2 = \frac{Q}{C_2}$$

The sum of voltages must be V, from which we obtain the charge Q

$$V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right), \qquad \rightarrow\; Q = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V = C_{eq}V, \qquad C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

Other useful forms are (take your pick)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}\,, \qquad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Together, the two series capacitors behave as a single capacitor C_{eq}.

Knowing Q, we find the voltages

$$V_1 = \frac{C_2}{C_1 + C_2}V\,, \qquad V_2 = \frac{C_1}{C_1 + C_2}V$$

Two capacitors C_1 and C_2 are connected in parallel with a battery V. The capacitors acquire the same voltage V. Their unknown charges are found directly

$$Q_1 = C_1 V\,, \qquad Q_2 = C_2 V$$

The total charge stored in the circuit is

$$Q = Q_1 + Q_2 = (C_1 + C_2)V = C_{eq}V, \qquad C_{eq} = C_1 + C_2$$

and as expected, the equivalent capacitance of the circuit is simply the sum of capacitances.

Equivalent Circuits. When you encounter two or more capacitors in series, you may replace them with their equivalent capacitor and simplify the circuit.

When you encounter two or more capacitors in parallel, you may replace them with their equivalent.

A little care is required when identifying sets of series and parallel capacitors. Pictures help.

Electric current is a “river” of charges flowing through the circuit. The same river flows through series capacitors, while different rivers flow through parallel capacitors.

Electric voltage is like height from which the river becomes a water fall. Parallel capacitors have the same height (same voltage), while series capacitors stack one atop the other so that the total height (voltage) is the sum of the individual heights.