Electric potential energy U and electric potential V. In mechanics we calculated the gravitational potential energy of a body, and were able to analyze the body's motion in a simple way.

Now, we calculate the electric potential energy of a body, which again enables us to analyze the motion.

To refresh our memories, the work done by a force acting on a body, and the change in the body's kinetic energy are related by the work energy theorem

$$K_i = \frac{m}{2} v_i^2, \qquad K_f = \frac{m}{2} v_f^2\,, \qquad W = F \Delta x\,, \qquad \Delta K = W$$

What if the force is not parallel to the displacement? Recall from mechanics the more general case of work done by a constant force $\vec{F}$ acting through a displacement $\vec{r}$, with angle θ between the vectors

$$W = \vec{F} \cdot \vec{r} = Fr\cos\theta$$

We defined the change in gravitational potential energy as the negative of the work done by the gravity field acting on a body of mass m through a displacement $h = y_f - y_i$

$$\Delta U_G = -W_G = mgh$$

The change in elevation h can be positive or negative, corresponding to the body moving up or down.

Defining the potential energy to be zero at $y = 0$ we have

$$U = mgy$$

The work done by a uniform electric field on a charge moving through displacement $\vec{s}$ is given by the product of the force and displacement

$$W_E = \vec{F}_E \cdot \vec{r} = q\vec{E} \cdot \vec{r} = qEr\cos\theta$$

Diagraming four cases, for $\pm q$ and for $\vec{E}$ parallel or anti-parallel to $\vec{r}$, we see that the work is positive or negative if the force and displacement are parallel $\rightrightarrows$ or anti-parallel $\rightleftarrows$.

The work done by the uniform field is independent of the particle's path. Diagraming and comparing different paths, where all paths have the same initial and final positions, hence the same displacement, we seen that the paths share the same parallel component of displacement $r\cos\theta$.

As the field works on a particle, the field exchanges energy with the particle. When the particle's kinetic energy increases the field's potential energy decreases; thus ΔK and ΔU are opposite in sign.

We define the change in electrical potential energy as the negative of the work done by the electric field on a body of charge q

$$\Delta U_E = -W_E = -q\vec{E} \cdot \vec{r}$$

Dividing by the charge q, we obtain the "electric potential" V and define a new unit, the $Volt$

$$\Delta V = \frac{\Delta U_E}{q} = -\vec{E} \cdot \vec{r}, \qquad \left[\frac{N \cdot m}{C} = \frac{J}{C} = Volt\right]$$

whose units are "energy per unit charge".

Take care to distinguish U, which has units of energy, from V.

You are all familiar with volts, as in "1.5V battery". What does that mean? If a charge of $q = +1C$ moves from the top of the battery to the bottom (say along a wire), it experiences a change in potential

$$\Delta V = V_{bat} = 1.5V$$

and a change in potential energy of

$$\Delta U_E = q\Delta V = 1.5\,J$$

Supposed you attach the battery to parallel plates, creating a uniform electric field that pushes an electron through the potential difference (we have to do this in vacuum). How fast will the electron be moving? Let $v_i = 0$

$$\Delta K = W_E = -\Delta U_E = -q\Delta V = -(-e)\Delta V = eV_{bat}, \qquad \Delta V = V_{bat}$$

$$\frac{m}{2} v_f^2 = eV_{bat}\,, \qquad v_f = \sqrt{2\frac{e}{m}V_{bat}} = 7.3 \times 10^5 \frac{m}{s}$$

If we discharge the 1.5V battery through a wire, does this mean the free electrons in the wire acquire such speed? No, because a wire is filled with atoms, it is an obstacle course; an electron bumps into atoms all along its way and loses energy, causing the atoms to vibrate more (heat up). By the time the electron reaches the bottom, its tongue is hanging out, it is tired and flops onto the ground motionless, having lost all of its energy $\Delta K = -\Delta U$ to heat.

If you are given the voltage (the potential) at points A and B, the potential difference

$$\Delta V = V_B - V_A\,, \qquad \Delta U = q\Delta V$$

can be used to find the change in potential energy experienced by a charge q as it moves from A to B, which can be used in the work-energy theorem $\Delta K = -\Delta U$ to analyze the motion of the charge.

If we define a zero reference at either the initial or final positions of the charge q, we write simply

$$V = -Er, \qquad U = qV$$

We can sketch the "equi-potential" lines of the uniform electric field. These lines are perpendicular to the field lines.