Last time we added two forces exerted on a charge by two other charges. Let's review it. Charge q_2 exerts a force on charge q_1. A unit vector $\hat{r}$ pointing from 2 to 1 is found using our formulas

$$\hat{r} = \frac{\vec{r}}{r}\,, \qquad \vec{r} = \Delta x\, \hat{x} + \Delta y\, \hat{y}\,, \qquad \Delta x = x_1 - x_2\,, \qquad \Delta y = y_1 - y_2\,, \qquad r = \sqrt{\Delta x^2 + \Delta y^2}$$

Our calculation went as follows

$$r_{21} = \sqrt{2^2 + 3^2} = \sqrt{13}\, m\,, \qquad \hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}} = \frac{-2}{\sqrt{13}}\hat{x} + \frac{-3}{\sqrt{13}}\hat{y}$$

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} = 0.768\hat{x} + 1.152\hat{y}\ \mu N$$

$$r_{31} = \sqrt{1^2 + 4^2} = \sqrt{17}\, m\,, \qquad \hat{r}_{31} = \frac{\vec{r}_{31}}{r_{31}} = \frac{1}{\sqrt{17}}\hat{x} - \frac{4}{\sqrt{17}}\hat{y}$$

$$\vec{F}_{31} = k\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} = 0.642\hat{x} - 2.57\hat{y}\ \mu N$$

$$\vec{F} = \vec{F}_{21} + \vec{F}_{31} = 1.410\hat{x} - 1.418\hat{y}\ \mu N$$

The electric field. The force created by a charge Q located at the origin on charge q located at $\vec{r}$ is

$$\vec{F} = k\frac{Qq}{r^2}\hat{r}$$

Let q be positive. Then for positive Q, the force points radially outward from Q, and goes as $1/r^2$. For negative Q, the force points radially inward to Q.

If we divide by q we obtain the electric field of Q at $\vec{r}$, which depends only Q and $\vec{r}$, the field point

$$\vec{E} = \frac{\vec{F}}{q} = k\frac{Q}{r^2}\hat{r}\ \frac{N}{C}$$

We call $\vec{E}$ the electric field of charge Q. We cannot see $\vec{E}$, but we can see its effects. A given field $\vec{E}$ exerts a force

$$\vec{F} = q\vec{E}$$

on a charge q immersed in the field.

Notice that $\vec{E}$ does not depend on q. $\vec{E}$ is determined by Q and the field point $\vec{r}$.

For example, a charge $Q = 1\mu C$ placed at the origin produces the field

$$\vec{E} = kQ\frac{\hat{r}}{r^2} = 9000\frac{\hat{r}}{r^2}\ \frac{N}{C}$$

E field superposition. Just as the force on charge q is the sum of forces due to a set of charges $\{Q_i\}$

$$\vec{F} = \sum \vec{F}_n = \vec{F}_1 + \vec{F}_2 + \cdots = kq\left[\frac{Q_1}{r_1^2}\hat{r}_1 + \frac{Q_2}{r_2^2}\hat{r}_2 + \cdots\right]$$

the field at a point is the sum of the fields of the charges

$$\vec{E} = \frac{\vec{F}}{q} = k\left[\frac{Q_1}{r_1^2}\hat{r}_1 + \frac{Q_2}{r_2^2}\hat{r}_2 + \cdots\right] = \vec{E}_1 + \vec{E}_2 + \cdots$$

The field of single point charge is the easiest to sketch. Sketch the fields of $+q$ and $-q$.

To sketch the field lines of two (or more) charges, sketch dash marks to show the line of $\vec{E}$ at a number of different points. Your eye will begin to see the shape of the field.

Let's sketch the field of an electric dipole (example 21.8 page 704).

You can play with some simple field configurations online at Caltech (**see my lecture notes**). Y&F Chap 21 also has some nice plots of single charges and pairs of charges of same and opposite sign.

My method: first sketch the near fields of each charge, ignoring the other charges.

A few rules become apparent:

Field lines originate on positive charges and terminate on negative charges (some of the charges may be far away, beyond the edges of the page or chalk board).

Field lines do not cross. The sum of superimposed fields at a point $\vec{r}$

$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \cdots$$

is a single field at that point.

The electric field vectors at each point in space are tangent to field lines.

The number of field lines emanating from a charge is proportional to the magnitude of the charge.

Recall that the value of k depends on choice of units of length, mass, time and charge. For simplicity, it is possible to choose units such that $k = 1$, then we write

$$E = \frac{Q}{r^2}$$

In lab you will work with this simple form of the field for ease of calculation.

The field (rather than the source charge) takes center stage. We can specify an electric field $\vec{E}$ without specifying the charges that create it. Suppose we have a uniform electric field in space, pointing in the $+y$ direction

$$\vec{E} = 100\,\frac{N}{C}\,\hat{y}\,, \qquad \textit{constant magnitude and direction}$$

What happens to an electron if you place it in a uniform field? The electron experiences a force of

$$\vec{F} = q\vec{E} = (-1.602 \times 10^{-19} C)\left(100\,\frac{N}{C}\,\hat{y}\right) = -1.602 \times 10^{-17} N\hat{y}$$

Notice that the force is in the negative y direction.

The electron experiences an acceleration (from Newton's 2nd law) that dwarfs $g = 9.81\ m/s^2$

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{e}{m}\vec{E} = \frac{-1.602 \times 10^{-17} N\hat{y}}{9.11 \times 10^{-31} kg} = 1.76 \times 10^{13}\,\frac{m}{s^2}$$

After accelerating through a distance $d = 0.01\ m$ the electron's speed is

$$v^2 = 2ad\,, \qquad \rightarrow \qquad v = \sqrt{2ad} = 5.9 \times 10^5\,\frac{m}{s}, \qquad d = \frac{a}{2}t^2\,, \qquad t = \sqrt{\frac{2d}{a}} = 34\ ns$$

It took $34\ ns$ for the electron to reach this speed. See how easy it is to work with the electric field?

What charge distribution creates a uniform electric field? Parallel plates of opposite charge!

Example 21.8 Electron Trajectory is obtained from the equations of motion for constant acceleration

$$x = v_0 t\,, \qquad y = \frac{a}{2}t^2 = -\frac{eE}{2m_e}t^2, \qquad y = -\frac{eE}{2m_e v_0^2}x^2$$

the equation of a parabola. What is the trajectory equation for a proton? Replace m_e with m_p and flip the sign of the charge! The field drives the electron downward and the proton upward.

The electron's motion reminds us of a ball rolling off of a table (sketch). The Earth's constant gravitational field is analogous to the electric field

$$\vec{g} = -g\hat{y}, \qquad \vec{F}_G = m\vec{g} \qquad \Leftrightarrow \qquad \vec{E} = E\hat{y}, \qquad \vec{F}_E = q\vec{E}$$

For gravity, multiply by mass to get the force. For electricity, multiply by charge to get the force. Both are cases of constant acceleration along the vertical line.

Show how parallel plates produce a uniform field. Pick any point between the plates, then pick two elements of charge on the same plate, one to the left of the point, the other to the right, sketch their fields and see how their fields add. Horizontal components cancel, while vertical components add. We use calculus to perform the exact calculation.

We covered topics in section Y&F 21.4 thru 21.6. Make sure you read these!