Electric Current, Current Density, Resistivity and Resistance. A voltage source such as a battery, a generator, or a solar cell, will make current to flow through a circuit. We say the battery applies an "electro-motive force", or "emf" for short. The textbook specifies a battery emf as

$$\mathcal{E} = 1.5V$$

The name "emf" is misleading; this is not a force, it is a voltage, pure and simple! The battery voltage creates an average electric field within the conductor of length L, and it is this field that exerts a force

$$F = qE = q\frac{V}{L}$$

on the free charged particles within the conductor, each of charge q, and pushes them through the wire. The charged particles collide with the atoms in the wire. The collisions produce an average "drag" force on the charged particles which opposes the electric force, so that the charged particles move with an average "drift" speed v_d through the wire. The motion is similar to a raindrop falling through air; the Earth provides the "gravito-motive force", and the air exerts drag; in equilibrium the forces cancel and the raindrop falls at constant speed.

You can pick a point on the wire and count the number of charged particles passing per second; this is current. If an amount of charge ΔQ passes in a time Δt the average current $\bar{I}$ is

$$\bar{I} = \frac{\Delta Q}{\Delta t}, \qquad I = \frac{dQ}{dt}, \qquad in\ units\ of\ \frac{Coulombs}{s} = Amperes$$

The latter form is the instantaneous current I, the current at a moment in time, an infinitesimal amount of charge passing in an infinitesimal amount of time.

We call the moving charged particles "free charges". We can count the number $\mathcal{N}$ of free charges in a volume $\mathcal{V}$. The density n of free charges is then

$$n = \frac{\mathcal{N}}{\mathcal{V}}, \qquad units\ of\ \frac{\#\ particles}{m^3}$$

Different materials have different free charge densities.

Consider a current carrying wire of cross section A. During a time Δt free charges move a distance

$$d = v_d \Delta t$$

The volume of charge that moves past a point is

$$\mathcal{V} = Ad = Av_d\Delta t$$

The number of free charges in the volume is

$$\mathcal{N} = n\mathcal{V} = nAv_d\Delta t$$

The amount of free charge in the volume is

$$\Delta Q = q\mathcal{N} = qnAv_d\Delta t$$

and the current is

$$\bar{I} = \frac{\Delta Q}{\Delta t} = qnAv_d \ , \qquad or \ \ I = \frac{dQ}{dt} = qnAv_d$$

Finally we define current density J to be

$$J = \frac{I}{A} = nqv_d \ , \qquad \vec{J} = nq\vec{v}_d \ , \qquad in\ units\ of\ \frac{Amperes}{m^2}$$

Example Y&F 25.1 calculates a typical current density and drift speed. A copper wire has diameter d, free electron density n, and carries current $I = 1.67\ A$.

$$d = 1.02mm, \qquad n = 8.5 \times 10^{28}\ \frac{e^-}{m^3}$$

The current density J and the drift velocity v_d are

$$A = \pi\left(\frac{d}{2}\right)^2 , \qquad J = \frac{I}{A} = 2.04 \times 10^6 \frac{A}{m^2} \ , \qquad v_d = \frac{J}{nq} = 0.15\ \frac{mm}{s}$$

Does the current density seem enormous? If our wire had a cross section of $1m^2$ it would carry 2 MA; thankfully our wire has a much smaller cross section.

Does the drift velocity seem more like the speed of a snail than an electron? For such a huge free electron density, even a small speed results in a mass migration, a sizable current.

Resistivity $\boldsymbol{\rho}$. An applied voltage V along a length of wire L creates an average electric field

$$E = \frac{V}{L}$$

If we compare wires made of different materials, say copper and aluminum, we find that for a given applied voltage, more current flows through the copper wire than the aluminum wire. We say the aluminum has a higher resistivity.

In a conductor the current density is proportional to the electric field

$$J \propto E$$

and is inversely proportional to the resistivity

$$J \propto \frac{1}{\rho}$$

Double the resistivity and halve the current density. From the above two relations we define resistivity

$$J = \frac{E}{\rho}, \qquad \rightarrow \qquad \rho = \frac{E}{J}, \qquad \sigma = \frac{1}{\rho}$$

We sometimes work with the conductivity σ, which is simple the reciprocal of resistivity. Think of permissive parents versus restrictive parents; the two measures are opposites.

In words, the resistivity is the ratio of electric field strength to current density; the resistivity tells you how strong E needs to be to produce a given J.

Resistance R. Writing the field as

$$E = \rho J, \qquad E = \frac{V}{L}, \qquad J = \frac{I}{A}, \qquad \rightarrow \quad \frac{V}{L} = \rho \frac{I}{A}, \qquad \rightarrow \; V = I\left(\frac{\rho L}{A}\right)$$

and substituting for E and J we obtain the formula for the "resistance" of a conductor in terms of its length and cross section

$$R = \frac{\rho L}{A}, \qquad in\ units\ of\ ohms\ \Omega$$

This formula is comparable to the formula for capacitance of parallel plates of area A and spacing d

$$C = \frac{\epsilon_0 A}{d}$$

In terms of R, the relationship between the voltage and current of a conductor is Ohm's Law

$$V = I\left(\frac{\rho L}{A}\right), \qquad \rightarrow \qquad V = IR, \qquad 1\Omega = \frac{1V}{1A}$$

Similar to the useful formula $Q = CV$ for capacitors, Ohm's Law is our useful formula for resistors.

Example Y&F 25.2 calculates a typical resistance. Take a length $L = 50m$ of the wire from the previous example. From Table 25.1 the resistivity of copper is

$$\rho = 1.72 \times 10^{-8}\ \frac{V \cdot m}{A} = \Omega \cdot m$$

The electric field strength E inside the wire, the potential difference V along the length of the wire, and the resistance R of the wire are

$$E = \rho J = \frac{\rho I}{A} = 0.035 \frac{V}{m}, \qquad V = EL = 1.75V, \qquad R = \frac{\rho L}{A} = 1.05\ \Omega$$

This is the simplest resistor circuit, a wire attached to a voltage source. The wire is the resistor.

Resistivity and resistance depend on temperature. In terms of the temperature coefficient α

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$