STUDY GUIDE 4: Equilibrium, Angular Kinematics, and Dynamics

Objectives

25. Define torque. Solve problems involving objects in static equilibrium.

26. Define angular displacement, angular velocity and angular acceleration. Given the graph or the functional form of one of the quantities versus time, determine the graphs of the other two. Describe in words and equations the motion from an analysis of one or more of the graphs.

27. Define moment of inertia, and solve problems involving rotational motion of rigid bodies subject to a net torque.

28. Calculate the angular momentum, relative to a specified axis, of a point mass traveling in a straight line.

29. Calculate the angular momentum of a rigid body whose angular velocity is specified.

30. Solve problems using the law of conservation of angular momentum.

Suggested Study Procedure for Chapter 12.

Study Secs. 11.2 and 12.1 through 12.3.
Answer Questions 10, 11, and 12.
Study Examples 12.1 through 12.4.
Do Problems 13, 21, 41, 43, 49, 51, and 61.

A. So far we've ignored ONE LITTLE DETAIL about motion: that objects can ROTATE as well as TRANSLATE. We'll TURN to that little detail in this last section of the course. Perhaps you will be happy to hear that we don't really need any new theory for this. We just need to recast all of the old familiar ideas into angular terms. That's one of the things we'll be illustrating and emphasizing a lot in lecture and conference meetings.

B. We think that it is preferable to consider STATIC EQUILIBRIUM first before letting objects rotate, so we're going to deviate just a bit from the order of topics in the text. Because of the timing of Lab Experiment #4, the material of Sec. 11.2 on TORQUE should already be familiar to you, and that's all you need as background to Chapt. 12. Note that the Problem-Solving Hints on p. 366 apply to ALL of the work covered in Sec. 12.3. About all that changes from one problem to the next is the geometry of the situation! This is what takes some practice.

Suggested Study Procedure for Chapter 10.

Study Secs. 10.1 through 10.8.
Answer Questions 3, 4, 14, 15 and 16.
Study particularly Examples 10.1, 10.2, 10.4, 10.5, 10.9, 10.10, and 10.12 through 10.15.
Do Problems 5, 7, 9, 13, 19, 23, 33, 39, 45, 53, 55, 59, and 71.

A. As you study Secs. 10.1 through 10.3, keep in mind that ALL of the equations for rotational motion with constant angular acceleration are identical in form to those of one dimensional motion with constant
linear acceleration. Table 10.1 is intended to help you learn the new set of angular symbols and to see how they are interrelated in constant acceleration situations.

B. MOMENT OF INERTIA plays a role in rotational motion similar to that of MASS in translational motion. We will emphasize the use of moment of inertia in rotational motion rather than the calculation of moment of inertia for different bodies. On the other hand, we will expect you to know the expression for the moment of inertia for our standard objects: the hoop, the uniform disk, the stick, and the point particle at distance R from the rotation axis.

C. Study Secs. 10.6 and 10.7 with particular care. These sections show how to apply Newton's laws to problems involving rotation. Notice that the Free-Body Diagram is still an indispensable part of our problem-solving strategy!

D. Prior to this time we have ignored or neglected the effects of pulleys in problems other than their role in changing the direction of a force. Examples 10.12, 10.13, and 10.15 show how a real pulley (with mass!) can affect a system.

Suggested Study Procedure for Chapter 11.

Study Secs. 11.3 through 11.5. Skim Sec. 11.1.
Answer Questions 4, 13, 14, 15, 20, and 21.
Study Examples 11.4 through 11.6 and 11.8 through 11.10.
Do Problems 19, 31, 33, 35, 37, 39, and 41.

A. ANGULAR MOMENTUM deserves more time than we can assign to it! Not only is it important in understanding all sorts of everyday situations but also in the study of phenomena from the sub-atomic way up through the astronomical. For now, however, we have to content ourselves with Secs. 11.3, 11.4, and 11.5, and Examples 11.4 through 11.10, paying particular attention to the definition of angular momentum of a point particle (Eqn. 11.16), the definition of angular momentum of a rigid body (Eqn. 11.21), and the principle of ANGULAR MOMENTUM CONSERVATION (as shown in Examples 11.8 through 11.10). Fortunately, this will be enough coverage to help you understand the inner workings of many everyday angular motion situations that previously puzzled you.

HOMEWORK ASSIGNMENTS FOR STUDY GUIDE 4.

Homework Assignment #13 – due in lecture Monday, Feb. 18.

HW #13-1: Prob. 12-50, EXCEPT consider the foot to be basically horizontal, with the heel just barely off the floor so that forces R and T are vertical and at right angles to the foot. Solve for the magnitudes of R and T required to keep the heel just barely off the floor. (You should be impressed!)

HW #13-2: Prob. 12-43, EXCEPT change the mass of the gate to 75.0 kg and the tension in the guy wire to 300 N.

HW #13-3: Prob. 12-44, EXCEPT change the boom angle from 65° to 50°, and the cable angle from 25° to 40°.

Homework Assignment #14 – due in lecture Wednesday, Feb. 20.

HW #14-1: A uniform ladder 10.0 m long rests against a frictionless vertical wall at a 55° angle up from the horizontal. The ladder weighs 400 N, and the coefficient of static friction
between the foot of the ladder and the ground is 0.500. A person weighing 800 N climbs slowly up the ladder.

a) **Calculate** the maximum frictional force that the ground can exert on the ladder at its lowest end.

b) **Determine** the actual frictional force between the ladder and ground when the person has climbed 4.00 m up the ladder from the bottom.

c) **Determine** how far up the ladder the person can climb before the ladder begins to slip along the ground.

**HW #14-2:** A flywheel subject to constant angular acceleration rotates through 210 rad in a 3.00-s interval, at the end of which it has an angular speed of 80.0 rad/s.

a) **Determine** the angular speed at the beginning of this 3-s interval.

b) **Determine** the magnitude of the angular acceleration.

The flywheel now begins to coast to a stop, again with an angular acceleration of constant value. Given that the angular speed drops to zero during the next 16.0 s,

c) **determine** the magnitude of the angular acceleration during this portion of the motion, and

d) **determine** the angle through which the flywheel turns (expressed both in terms of radians and of revolutions) during this final 16 s.

**HW #14-3:** A Honda Prelude of mass 1350 kg starts from rest and speeds up with a constant tangential acceleration of 1.50 m/s² on a circular test track of radius 60.0 m. Treat the car as a particle.

a) **Calculate** the angular acceleration.

b) **Determine** the car’s angular speed 8.00 s after it starts.

c) **Determine** the car’s radial acceleration at this 8-s point.

d) **Sketch** a view from above showing the circular track, the car, the velocity vector, and the acceleration component vectors 8.00 s after the car starts.

e) **Determine** the magnitudes of the total acceleration and net applied force at this 8-s point.

f) **Calculate** the angle between the total acceleration vector (or net force vector) and the car’s velocity vector at the 8-s point.

g) **Comment** briefly on what will eventually happen in this situation as the elapsed time increases beyond the 8-s mark.

Homework Assignment #15 – due in lecture Friday, Feb. 22.

**HW #15-1:** A uniform solid disk of mass 5.00 kg and radius 0.400 m is pivoted about a horizontal axis through its center, and a small object of mass 3.00 kg is attached to the rim of the disk.

a) Given that the disk is released from rest with the small object at the end of a horizontal radius, **find** the angular velocity of the system when the small object is at the bottom.

b) Now imagine releasing the system, again with the point mass on a horizontal radius, but this time with an initial angular speed of 6.00 rad/s. **Show** that this is sufficient angular speed to enable the point mass to coast over the top of its circular path, and **calculate** the angular speed of the system when the point mass is exactly at the top of its circular path.

**HW #15-2:** A 15.0-kg disk with radius \( R = 0.200 \) m is mounted on a fixed, horizontal, frictionless axle. A 3.00-kg mass hangs from a very light string wrapped many times around the circumference of the disk. The system is released from rest and the 3-kg mass unwinds through a vertical distance of 2.50 m. For the system at this 2.50-m point, **calculate** by use of energy techniques (a) the speed of the 3-kg mass, (b) the angular
speed of the disk, (c) the kinetic energy of the disk. Also calculate (d) the angular acceleration of the disk as the string unwound through 2.50 m.

HW #15-3: Same situation as shown in Figure P10-43, EXCEPT the values are as follows: the pulley has a radius of 0.140 m and a moment of inertia of 0.300 kg•m², \( m_1 = 4.00 \text{ kg} \), and \( m_2 = 2.00 \text{ kg} \). Use energy techniques to calculate the speed of the 4-kg mass after it has descended a distance of 5.00 m, starting from rest. Using the speed result, next calculate the acceleration of the 4-kg mass and the cord tension on each side of the pulley.

Homework Assignment #16 – due in lecture Monday, Feb. 25.

HW #16-1: A 60.0-kg grindstone is 0.600 m in diameter and has a moment of inertia about the rotation axis of 2.70 kg•m². A knife is pressed down on the rim with a normal force of 50.0 N. The coefficient of kinetic friction between the blade and the stone is 0.60, and there is a constant friction torque of 5.00 N•m between the axle of the stone and its bearings.
   a) Calculate the amount of force that must be applied tangentially at the end of a crank handle 0.500 m long to bring the stone from rest to 120 rev/min in 9.00 s.
   b) Determine the tangential force needed at the end of the handle to maintain a constant angular speed of 120 rev/min.
   c) Determine the amount of time it takes for the grindstone to come to rest once the knife and the handle force are removed.

HW #16-2: A large turntable rotates about a fixed vertical axis, making one revolution in 6.00 s. The moment of inertia of the turntable about this axis is 1200 kg•m². A man of mass 80.0 kg is initially standing at the center of the turntable.
   a) Calculate the kinetic energy of the system (turntable plus man) when the man is at the center.
   b) Calculate the angular speed and kinetic energy the turntable would have if the man moves to a point on the rim of the turntable, 4.00 m from the center. Calculate also the man’s kinetic energy (from a ground-based point of view) when he is standing stationary at this point on the rotating turntable.
   c) The man now hops radially outward from the rim of the turntable, landing on the ground. Calculate the angular speed and kinetic energy the turntable has after the hop.
   d) Another 80.0-kg man who has been patiently standing on the ground near the edge of the turntable now hops onto the turntable, directing his hop toward the center and landing right on the rim, 4.00 m from the center. Calculate the angular speed and kinetic energy the turntable has after this new event.
   e) This second man now walks to the center of the turntable. Calculate the angular speed and kinetic energy the turntable has once the man reaches the center.

HW #16-3: You are assigned the design of a simple system to measure the speed of a bullet. You decide to shoot the bullet into a 10.0-kg solid disk of wood with radius 0.300 m mounted on a frictionless axle through its center and perpendicular to its face. The axle is vertical, so the disk lies in a horizontal plane. To see whether your idea is reasonable, you calculate the final rotational period of the disk if you shoot a 6.00-g (that’s gram) bullet at 360 m/s from a rifle along a line 0.250 m to the right of the center of the disk. How much time will it take your disk to make one revolution after the bullet has plowed into and stopped in the disk?