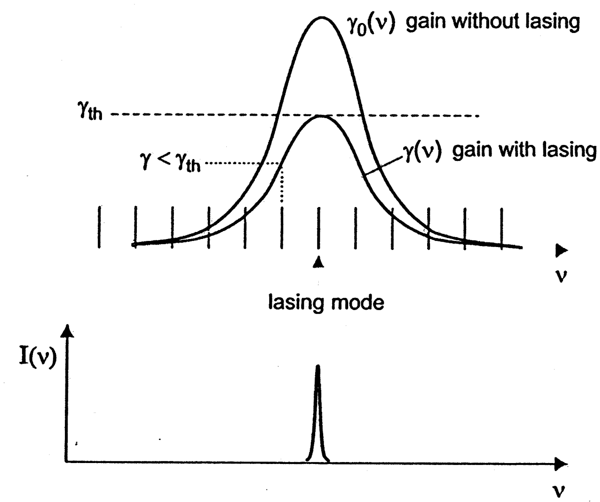
# Mode Spectrum of Laser Light

## Single mode operation

* homogeneous broadening

and

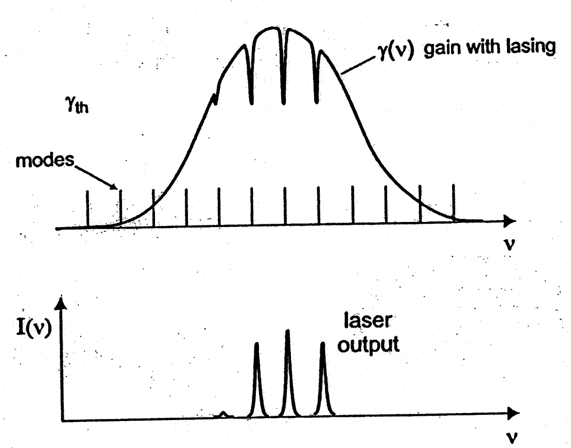
* uniform light intensity




## Multimode operation

Results with either 1) inhomogeneous broadening
or 2) non-uniform mode intensity

1) Inhomogeneous broadening (spectral hole burning)

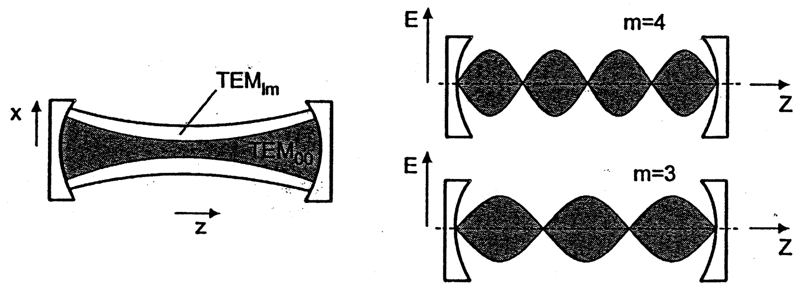



* Doppler effect in gases

$$\nu_0' = \nu_0\left(1 \pm \frac{v_z}{c}\right)$$

* different sites for ion in solid (Stark effect)

2) Non-uniform mode intensity
(spatial hole burning)



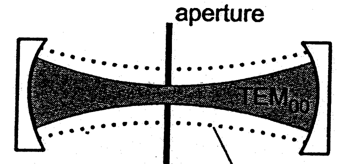

transverse modes

longitudinal modes

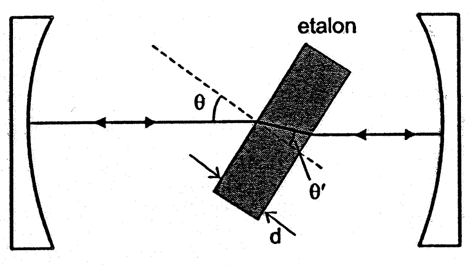
These cause inefficient use of pump energy

<u>Acheiving single-mode operation</u>:

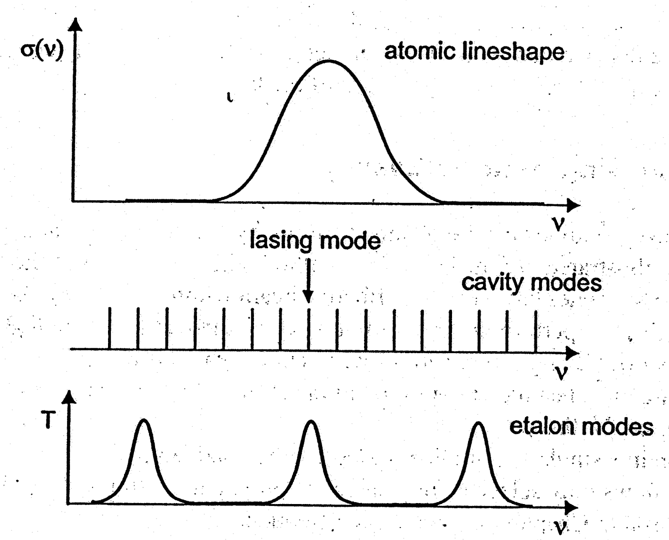
1) Single transverse mode:



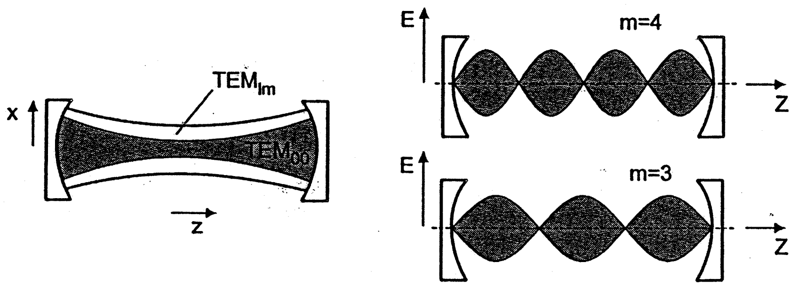

2) Single longitudinal mode:




$$\delta\nu_{etalon} = \frac{c}{2nd\cos\theta'}$$



2) Non-uniform mode intensity
(spatial hole burning)




transverse modes

longitudinal modes

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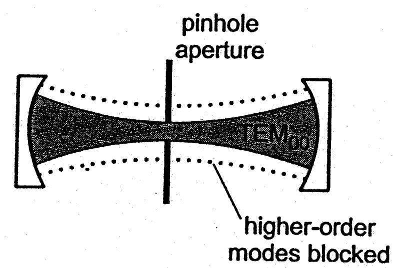
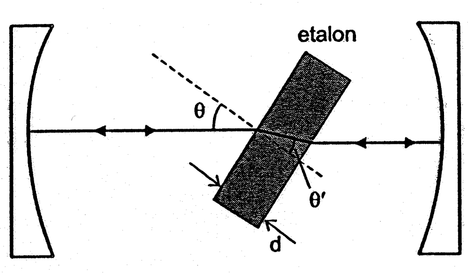



2) Single longitudinal mode:

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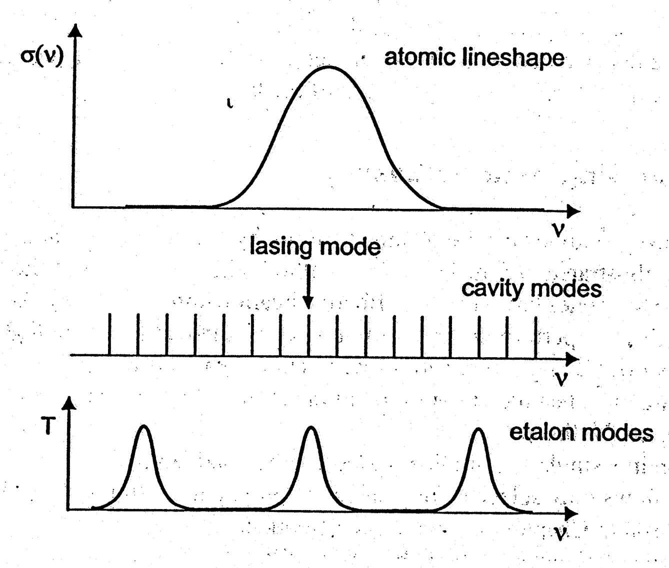
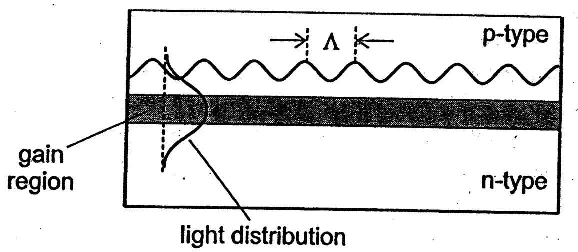


## Distributed feedback (DFB) laser



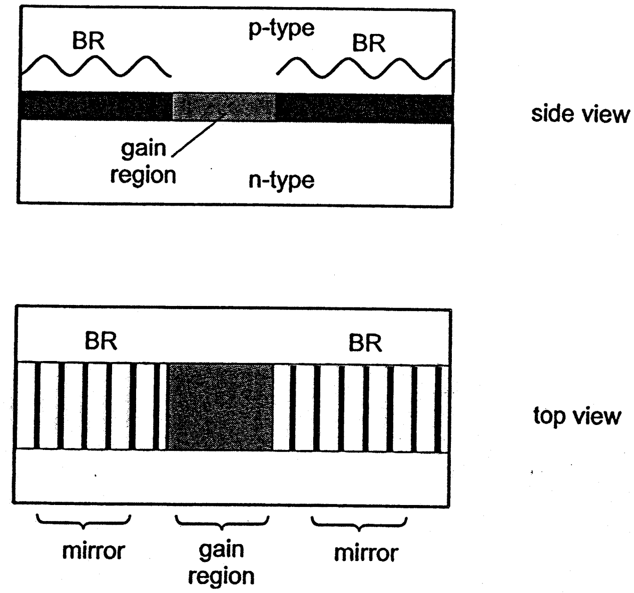

* high reflection for

$$\Lambda = \frac{\lambda}{2n} \cdot m$$

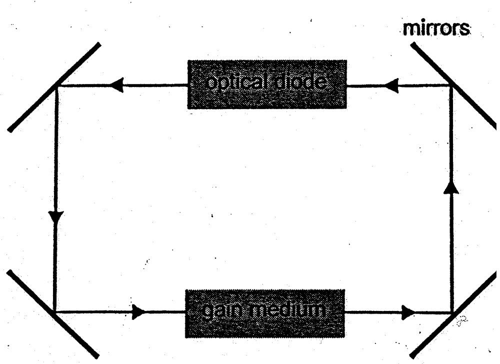
usually $m = 1$

* can temperature tune with $n(T)$

## Distributed Bragg grating (DBG) laser




## Ring laser




* no standing waves
* efficient utilization of pump energy

# Frequency stabilization

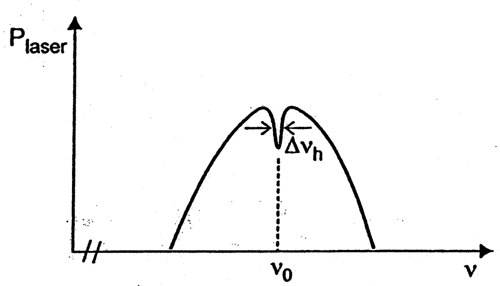
$$\nu_m = m \frac{c}{2nL}$$

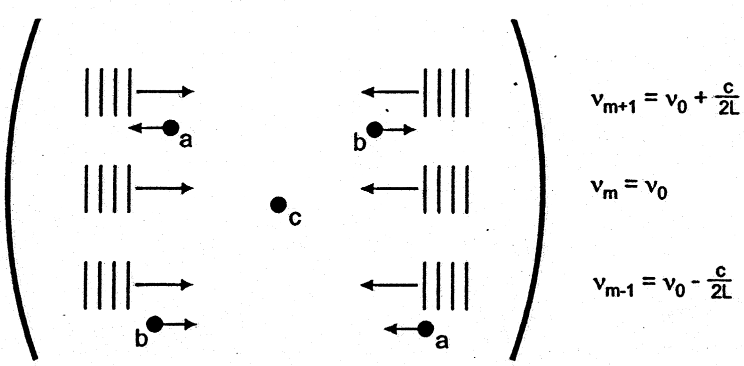
* $L$ varies with temp
* $n$ varies with temp
* "mode sweeping"

Solutions:

1) Temperature controller
2) Active stabilization with atomic resonance

Lamb dip

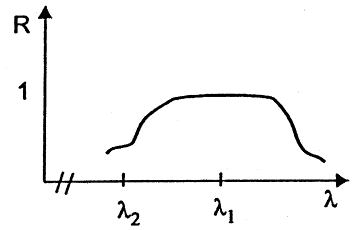


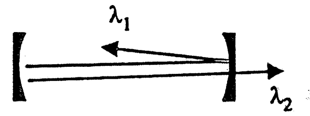




Atoms with zero velocity along axis interact with both counterpropagating travelling waves.

# Tuning the laser wavelength

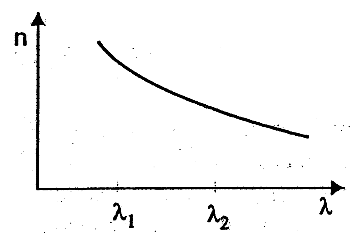
## 1. Spectrally selective mirrors

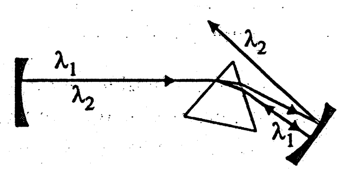



* good when $\lambda_1$ and $\lambda_2$ widely separated
(He-Ne, Nd:YAG)

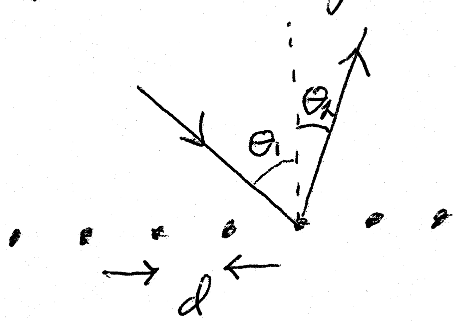
## 2. Intracavity prism

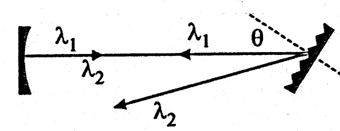



* good for low gain closely spaced $\lambda_1$ and $\lambda_2$
(e.g. Argon ion laser)

## 3. Diffraction grating




$$d(\sin\theta_1 - \sin\theta_2) = m\lambda$$

$m = 0, \pm 1, \pm 2, \ldots$ (order)

Retro-reflection: $\theta_2 = -\theta_1$

$2d\sin\theta_1 = m\lambda$ Bragg reflection

* Tune $\lambda$ by changing $\theta$
* Good for high gain lasers (inherent losses)
(e.g. $CO_2$, dye laser)

# Pulsed Laser Behavior

Desirable:
* pulsed pump
* Q-switching
* mode-locking

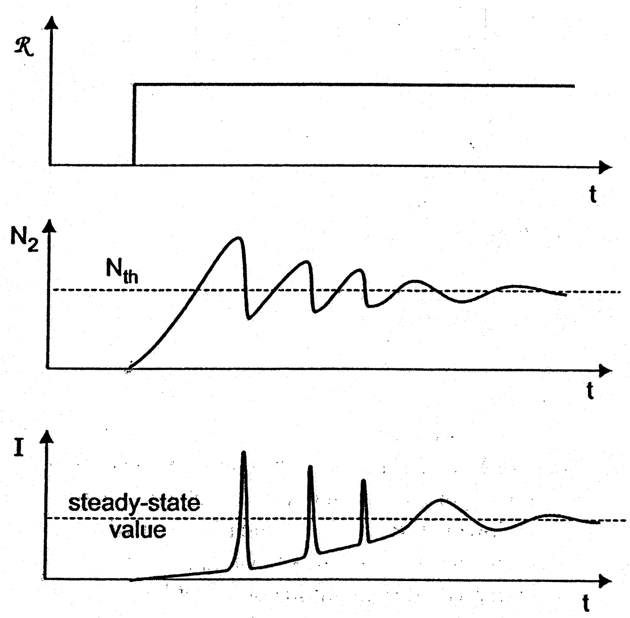
Undesirable: spiking

Basic equations:

$$\frac{dN_2}{dt} = R - N_2\left(\frac{I\sigma}{h\nu} + \frac{1}{\tau_2}\right) \qquad (1)$$

$$\frac{dI}{dt} = \left(c\sigma N_2 - \frac{1}{\tau_c}\right) I \qquad (2)$$

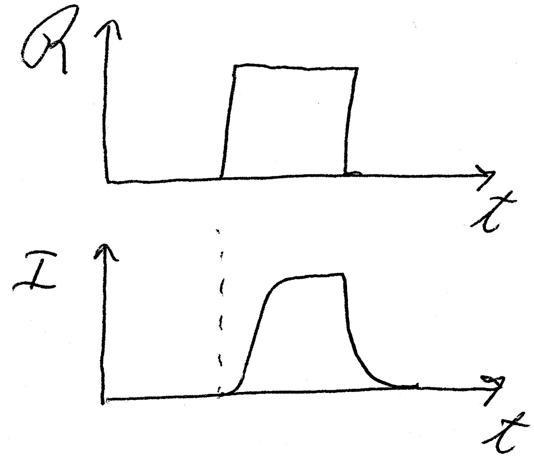
For a step input pump




Some lasers (e.g. ruby) do not reach steady-state, but instead exhibit <u>spiking</u>, a series of randomly spaced pulses.

Time scale 1–10 μs

## Pulsed pump

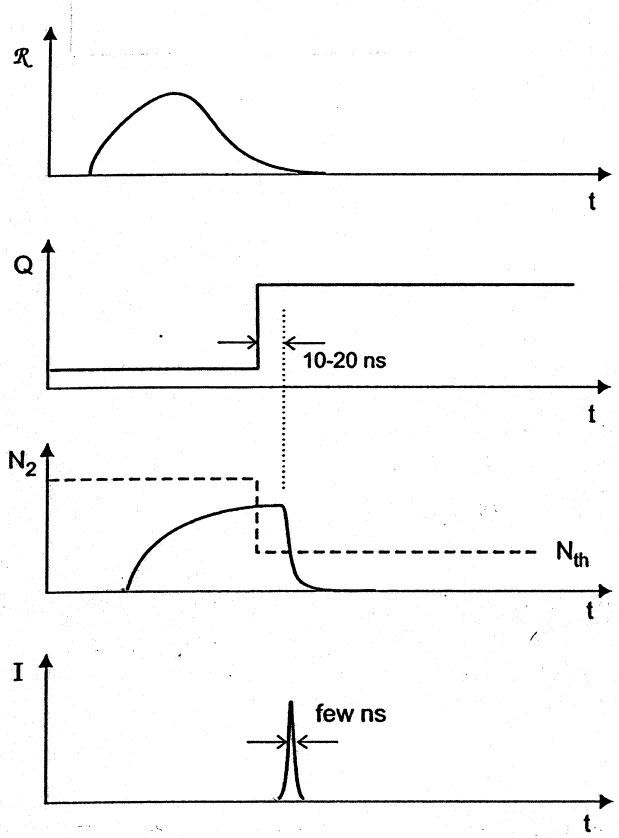



* common for semiconductor lasers
* difficult to get short pulses with optical excitation
* often used in combination with Q-switching and mode-locking

## Q-switching

$$Q = \frac{\nu_0}{\Delta\nu_{1/2}} = 2\pi \nu_0 \tau_c$$

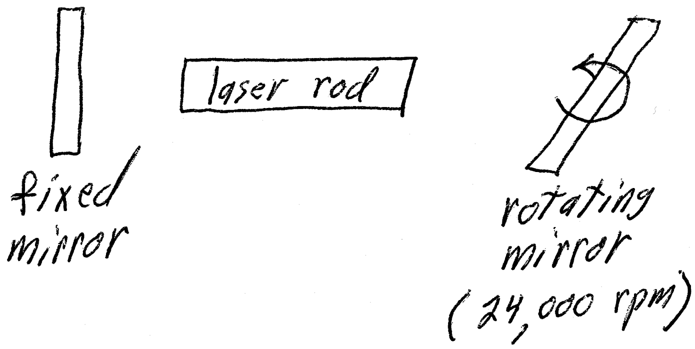
$$N_{th} = \frac{1}{c\sigma\tau_c} = \frac{2\pi\nu_0}{c\sigma Q}$$




* Must switch Q within ~10 ns
* stored energy is quickly channeled into single pulse.
* pulse width ~ few ns
* can also repetitively Q-switch with cw pump.

# Methods of Q-switching

## 1. Rotating mirror




$$\Delta\theta \simeq \frac{d}{L} \approx 10^{-3}\ \text{rad}$$

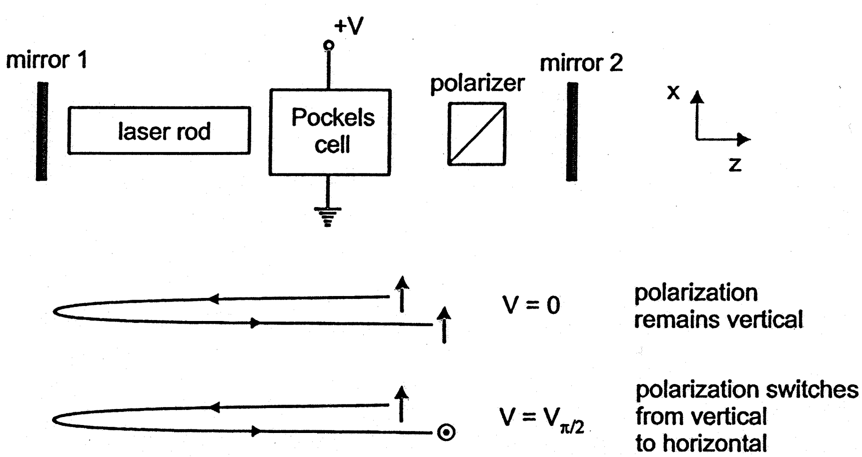
$$\omega t_{sw} = \Delta\theta$$

$$t_{sw} = \frac{10^{-3}\ \text{rad}}{2500\ \text{rad/s}}$$

$$t_{sw} = 400\ \text{ns}$$

"slow" switch ($\mu$s time scale)

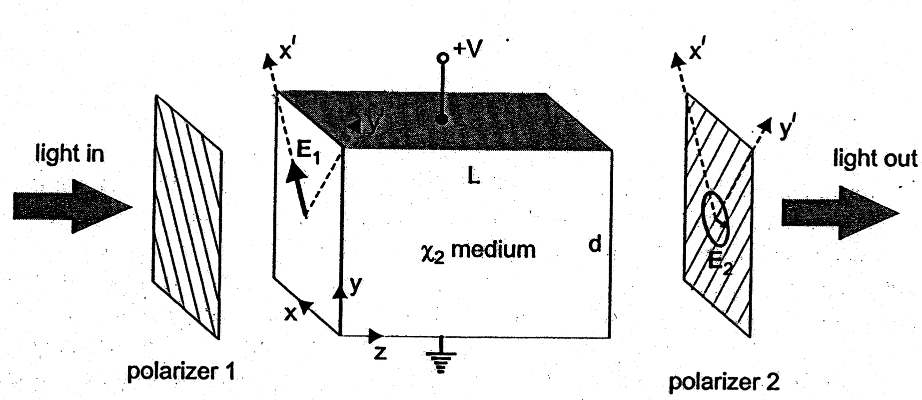
## 2. Electro-optic shutter



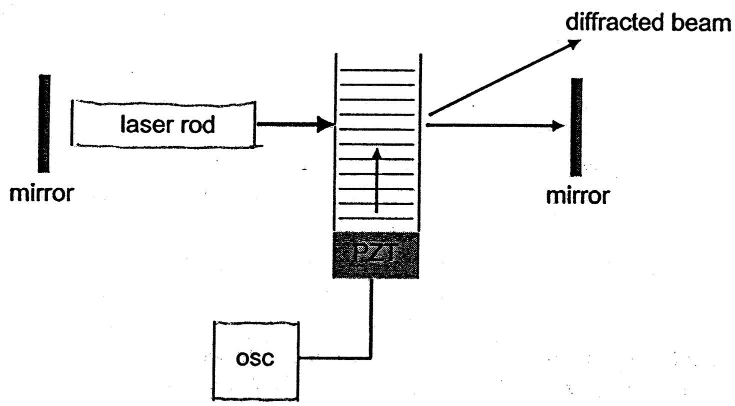

* Pockels effect-induced birefringence

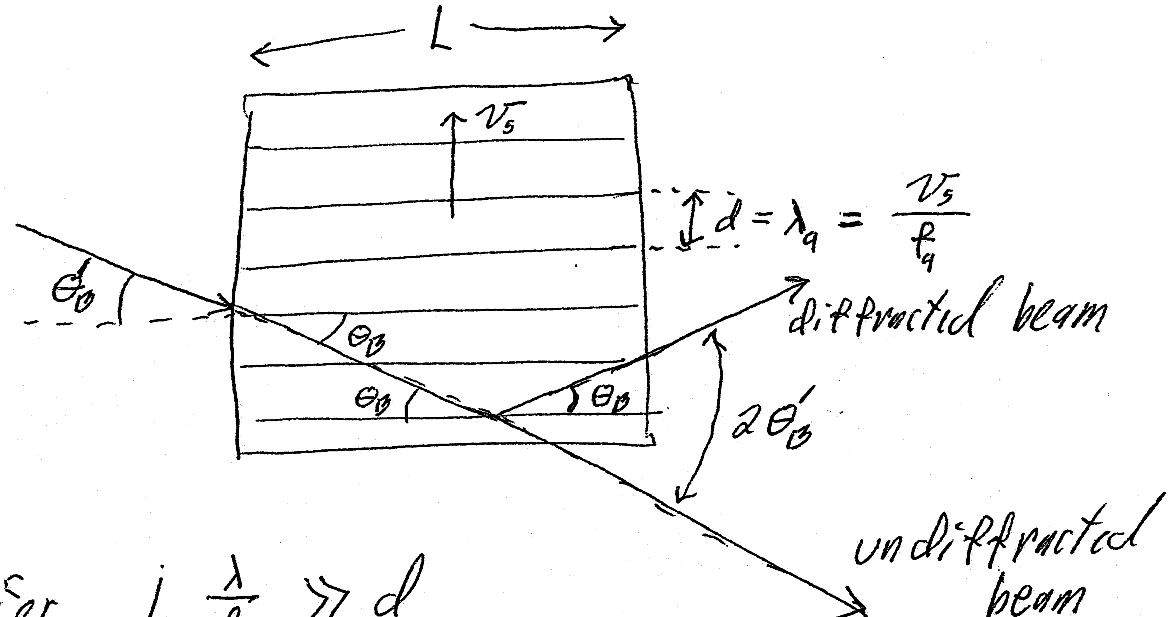
* Need high voltage (kV)

* can be fast ($\lesssim$ 20 ns)



## 3. Acoustooptic shutter




for $L \frac{\lambda}{d} \gg d$

have Bragg condition

$$\theta_B = \frac{\lambda/n}{2d} \qquad \text{(small } \theta_B\text{)}$$

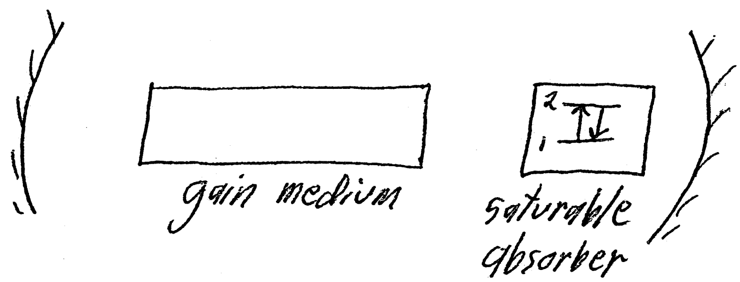
For small $\theta$, using Snell's Law, $\theta_B' = n\,\theta_B$

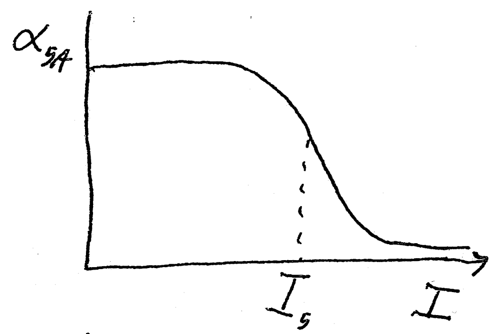
So

$$\boxed{\theta_B' = \frac{\lambda}{2d}}$$

$\lambda$ is free-space wavelength

## Passive Q-Switching



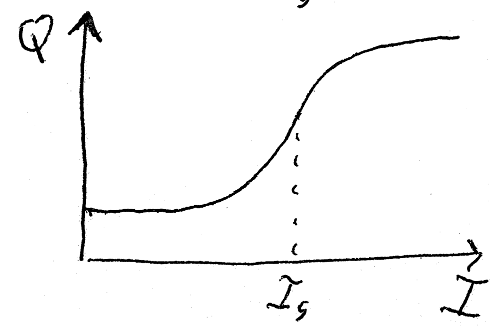




loss in saturable absorber

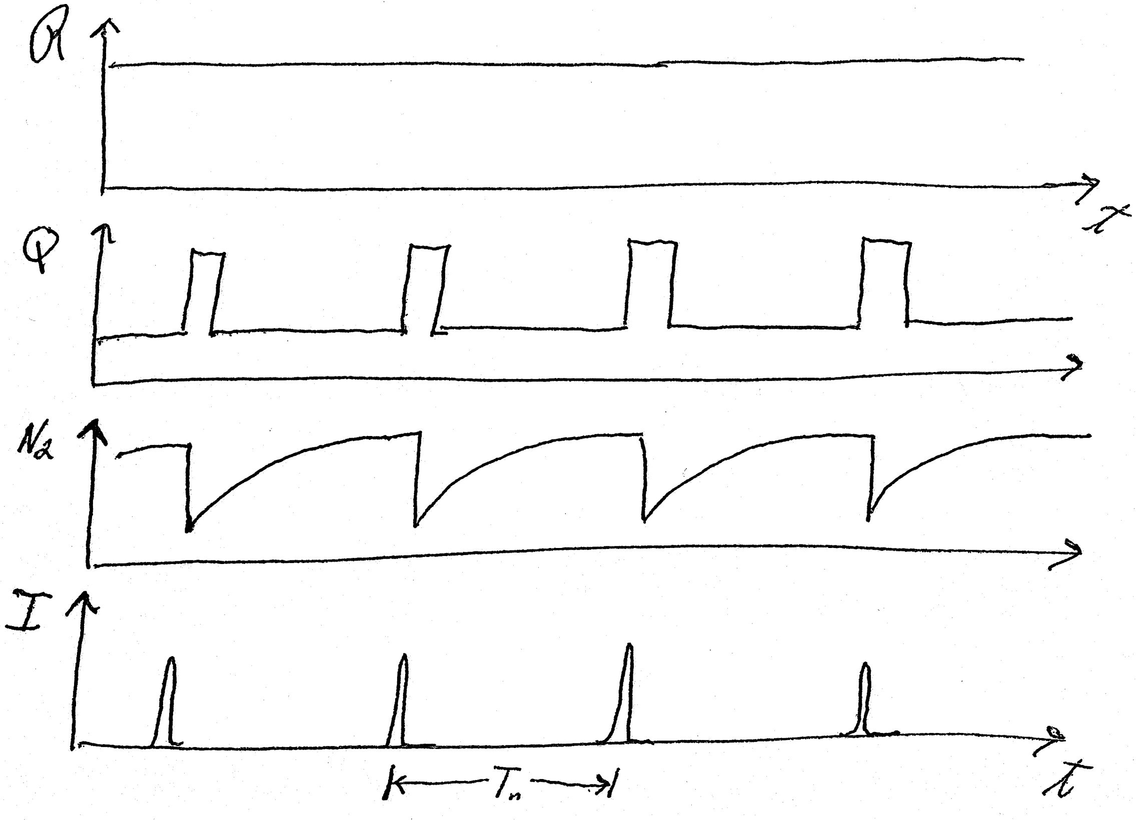
$$\alpha_{SA} = (N_1 - N_2)\,\sigma_{SA}$$

As $I$ increases, $(N_1 - N_2)$ decreases

$$\alpha_{SA} = \frac{\alpha_0}{1 + I/I_s}$$




## Repetitive Q-Switching

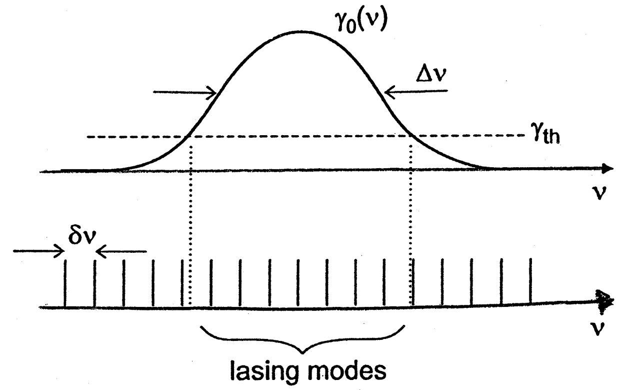



optimum

$T_p \simeq \tau_2$

$\Delta t_p \sim$ few ns

# Mode Locking




* CW lasing
* many modes lasing
* modes "locked" in phase
* very short pulses possible (ps or fs)

$\begin{cases} \delta\nu = \text{mode spacing} \\ \Delta\nu = \text{linewidth} \end{cases}$

At twice threshold,

$$N = \frac{\Delta\nu}{\delta\nu} \quad \# \text{ lasing modes}$$

If all N modes have same phase,

$$E(t) = \sum_m E_m e^{i\omega_m t}$$

Take

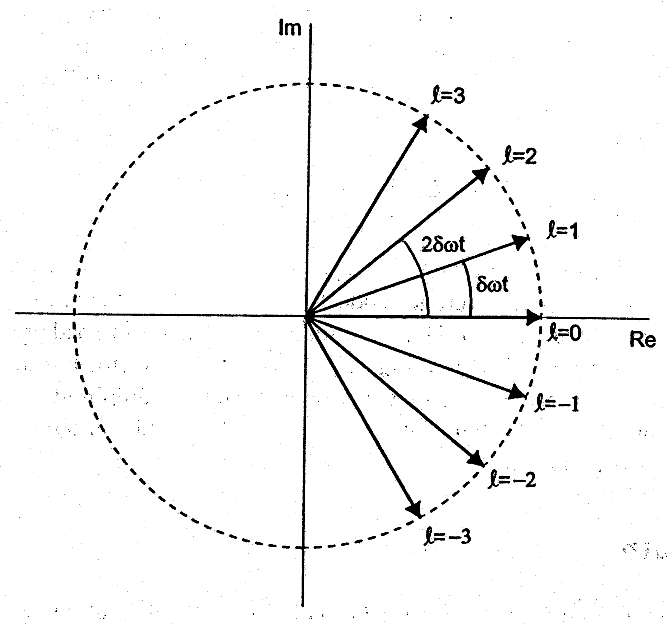
$$E_m = \begin{cases} E_0 & \text{for N lasing modes} \\ 0 & \text{all other modes} \end{cases}$$

Let $\omega_m = \omega_0 + \ell \cdot \delta\omega \qquad \ell = 0, \pm 1, \pm 2, \ldots \pm \frac{(N-1)}{2}$

Then

$$E(t) = E_0 e^{i\omega_0 t} \sum_{\ell=-N/2}^{N/2} e^{i\ell(\delta\omega t)}$$

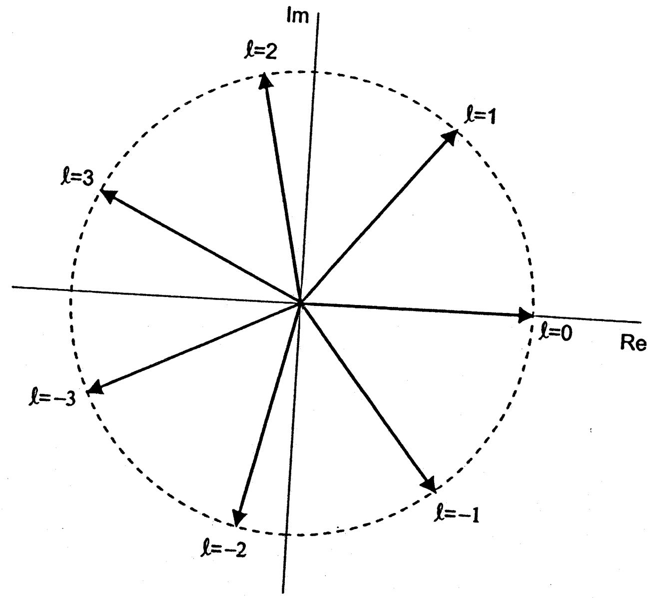
Phasors fan out as $(\delta\omega t)$ increases



At $t = 0$, phasors in line $\Rightarrow E = NE_0$

At $\frac{N}{2}\delta\omega t = \pi$, phasors cancel $\Rightarrow E = 0$

Light pulse has duration




$$\frac{N}{2}\delta\omega\,\Delta t_p = \pi$$

$$\Delta t_p \cong \frac{2\pi}{N\delta\omega} = \frac{1}{N\delta\nu}$$

and using

$$N \cong \frac{\Delta\nu}{\delta\nu}$$

then

$$\boxed{\Delta t_p \cong \frac{1}{\Delta\nu}}$$

* uncertainty relation
* transform limited pulse
* need wide bandwidth for short pulses

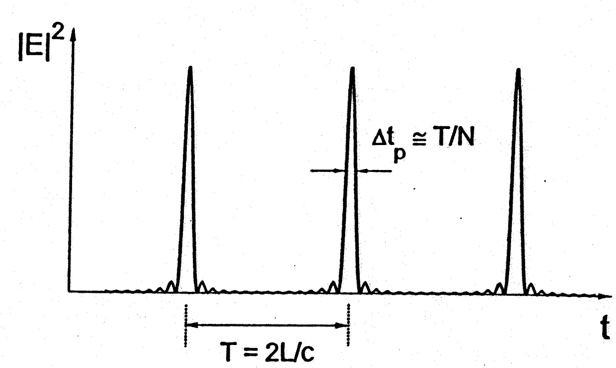
At $\delta\omega t = 2\pi$, all phasors in line again: get another pulse

$$T = \frac{2\pi}{\delta\omega} = \frac{1}{\delta\nu} = \frac{2L}{c}$$ repetition time (also cavity round-trip time)

Pulse width can be written

$$\Delta t_p = \frac{1}{N\delta\nu} = \frac{T}{N}$$

$$\boxed{\frac{\Delta t_p}{T} = \frac{1}{N}}$$



Example:

Fiber laser
$L \approx 5$ m
$\lambda = 1550$ nm
$\Delta\lambda = 50$ nm

Find
a) # lasing modes
b) $\Delta t_p$

Sol'n: a) $\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda = \frac{3\cdot10^8 \text{ m/s}}{1.55\cdot10^{-6}\text{ m}}\cdot\left(\frac{50 \text{ nm}}{1550 \text{ nm}}\right)$

$\Delta\nu = 3.84\cdot10^{12}$ Hz

$\delta\nu = \frac{c}{2nL} = \frac{3\cdot10^8 \text{ m/s}}{2(1.5)(5)\text{ m}} = 2.0\cdot10^7$ Hz $= 20$ MHz

$N = \frac{\Delta\nu}{\delta\nu} = \frac{3.84\cdot10^{12}}{2\cdot10^7} = \boxed{1.92\cdot10^5}$

b) $\Delta t_p = \frac{1}{\Delta\nu} = \frac{1}{3.84\cdot10^{12}} = 2.60\cdot10^{-13}$ s $= \boxed{260 \text{ fs}}$

## Energy in mode-locked pulse

* Mode-locking doesn't change average power
  Only re-distributes power in time

* Evaluate total energy in one round-trip time
  for both cw (non mode-locked) and pulsed:

$$P_{cw}\cdot T = P_{ML}\cdot\Delta t_p = P_{ML}\cdot\frac{T}{N}$$

$$\therefore \quad \boxed{P_{ML} = N P_{cw}}$$

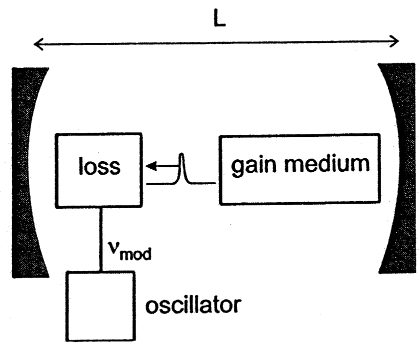
* Peak power greatly enhanced in mode-locking
  Useful for nonlinear optics applications

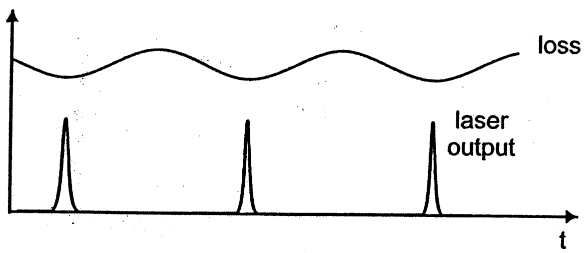
# Methods for mode locking

## (1) Active mode locking

set $\nu_{mod} = \frac{1}{T}$

"shutter" opens when pulse arrives

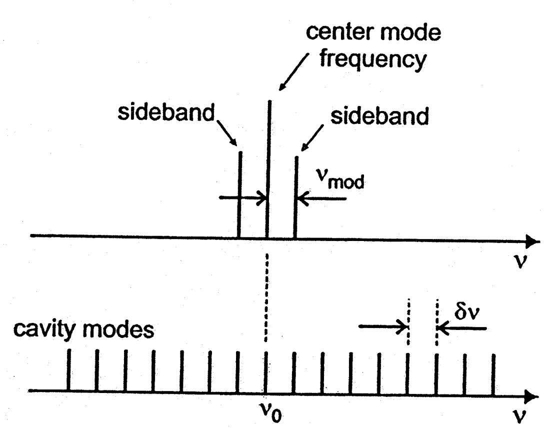



For $L = 1\ m$

$$\delta\nu = \frac{c}{2L} = 1.5 \cdot 10^{8}\ Hz$$
$$= 150\ MHz$$

Need stable RF oscillator

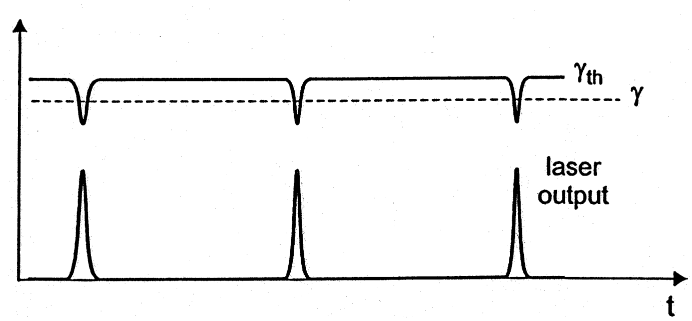


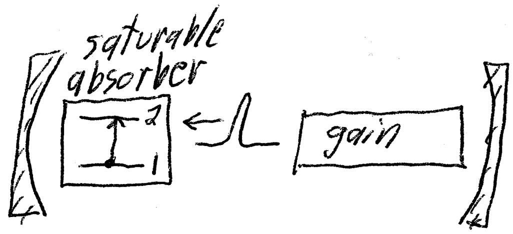

* In frequency domain, modulation creates sidebands that couple modes and lock the phases

## (2) Passive mode locking

* similar to Q-switching use saturable absorber

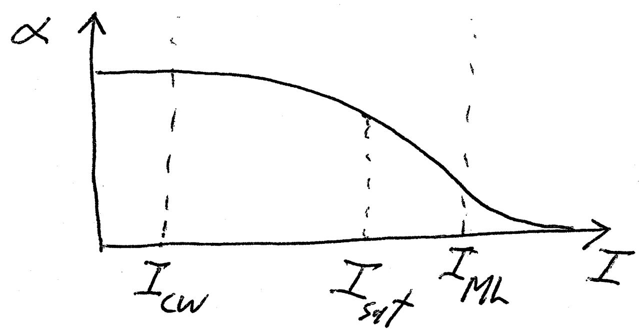
* set $\gamma < \gamma_{th}$ for no mode locking



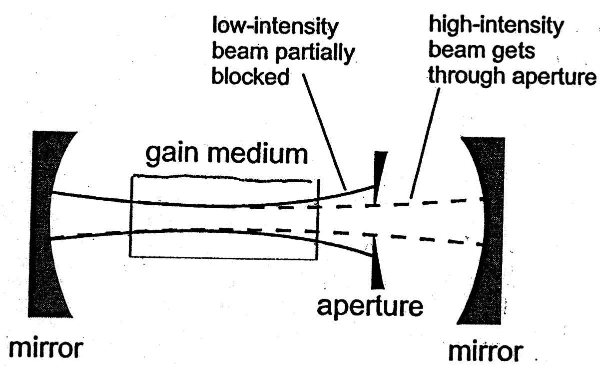



Mode-locked intensity is high enough to saturate absorption and bring $\gamma_{th} < \gamma$

$$I_{ML} = N I_{cw}$$




Alternative is Kerr lens mode locking:




* self-focusing for high intensity pulse
* fast time response due to nonlinearity of electron cloud position

## Time scales in pulsed lasers:

1. Pump pulse ~ ms
2. Spiking ~ μs
3. Q-switching ~ ns
4. Mode locking ~ ps