19.1 (a) For a homogeneously broadened system, the gain is reduced by a factor of two when the signal intensity equals the saturation intensity, given by

$$I_s = \frac{h\nu}{\sigma\tau_2} = \frac{hc}{\lambda\sigma\tau_2} = \frac{(6.63\times10^{-34})(3\times10^{8})}{(5.5\times10^{-7})(4\times10^{-20})(3\times10^{-9})} = 3.01\times10^{9}\ \mathrm{W/m^2}$$

(b) For Nd:glass, the saturation intensity is

$$I_s = \frac{(6.63\times10^{-34})(3\times10^{8})}{(1.054\times10^{-6})(4\times10^{-24})(290\times10^{-6})} = 1.63\times10^{8}\ \mathrm{W/m^2}$$

If the transition is inhomogeneously broadened, the gain is reduced by a factor of two when the signal intensity is given by

$$\frac{1}{\sqrt{1+I/I_s}} = \frac{1}{2}$$

$$1+\frac{I}{I_s} = 4$$

$$I = 3I_s = 4.88\times10^{8}\ \mathrm{W/m^2}$$

19.7 Assuming uniform excitation, the gain in a length $L = 15$ m is (1.5 dB/m)(15 m) = 22.5 dB. Using Eq.(19-16), the unsaturated gain coefficient is found from

$$\gamma_0 L = \frac{22.5}{4.34} = 5.18$$

The gain of the amplifier is $G = (5\times10^5)/(2.5\times10^4) = 20$, and solving Eq.(19-27) for I_s then gives

$$I_s = \frac{I_1(G-1)}{\gamma_0 L - \ln G} = \frac{(2.5\times10^4)(19)}{5.18-\ln(20)} = 2.17\times10^{5}\ \mathrm{W/cm^2}$$

20.5 (a) The threshold gain coefficient is

$$\gamma_{th} = \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) = \frac{1}{2(0.2)}\ln\left(\frac{1}{[0.998][0.99]}\right) = 0.030\ \mathrm{m}^{-1}$$

and the threshold population inversion is therefore

$$N_{2,th} = \frac{1}{c\sigma\tau_c} = \frac{\gamma_{th}}{\sigma} = \frac{3\times10^{-4}\ \mathrm{cm}^{-1}}{30\times10^{-14}\ \mathrm{cm}^2} = 1\times10^{9}\ \mathrm{cm}^{-3}$$

The fraction of Ne atoms that need to be excited is therefore

$$\mathrm{frac} = \frac{1\times10^{9}}{1.2\times10^{16}} = 8.3\times10^{-8}$$

(b) The number of atoms pumped to level 2 per unit time is $\mathcal{R}_{th}V$, where V is the volume of the gain medium. Using Eq.(20-18) we have

$$\mathcal{R}_{th}V = \frac{N_{2,th}V}{\tau_2} = \frac{(1\times10^{9}\ \mathrm{cm}^{-3})(20\ \mathrm{cm})\pi(5\times10^{-2}\ \mathrm{cm})^2}{1\times10^{-7}\ \mathrm{s}} = 1.57\times10^{15}\ \mathrm{s}^{-1}$$

(c) The minimum pump power required is

$$P_{in} = (1.57\times10^{15}\ \mathrm{s}^{-1})(20\ \mathrm{eV})(1.6\times10^{-19}\ \mathrm{J/eV}) = 5\times10^{-3}\ \mathrm{W}$$

In practice, the electrical pumping power required is much larger, on the order of a few Watts, due to inefficient excitation of the upper laser level.

20.6 From Eq.(20-21), the light intensity inside the laser cavity is

$$I = \frac{2P_{out}}{AT} = \frac{2(1.5 \times 10^{-3})}{\pi(5 \times 10^{-4})^2(0.01)} = 3.82 \times 10^5 \text{ W/m}^2$$

The photon energy is

$$h\nu = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19} \text{ J}$$

The induced emission rate is therefore

$$W_{21}^{ind} = \frac{(3.82 \times 10^5)(3 \times 10^{-17})}{3.14 \times 10^{-19}} = 3.65 \times 10^7 \text{ s}^{-1}$$

The spontaneous emission rate is simply the inverse of the radiative lifetime,

$$W_{21}^{spont} = 1/(30 \times 10^{-9}) = 3.33 \times 10^7 \text{ s}^{-1}$$

The ratio is then $W_{21}^{ind}/W_{21}^{spont} = 3.65/3.33 = 1.1$

20.11 The condition that the pump be three times threshold is

$$\frac{P_{in}}{P_{th}} = \frac{\gamma_0\, 2L}{\delta + T} = 3$$

Eq.(20-32) then becomes

$$4 \text{ W} = \frac{1}{2} A T I_s[3 - 1]$$

Solving for I_s then gives

$$I_s = \frac{2(4)}{AT[3-1]} = \frac{4}{\pi(5 \times 10^{-4})^2(0.005)} = 1.02 \times 10^9 \text{ W/m}^2$$