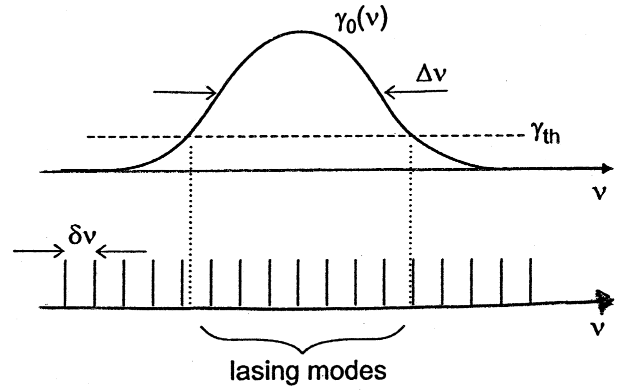
Mode Locking

* CW lasing
* many modes lasing
* modes "locked" in phase
* very short pulses possible (ps or fs)



$$\begin{cases} \delta\nu = \text{mode spacing} \\ \Delta\nu = \text{linewidth} \end{cases}$$

If all N modes have same phase,

$$E(t) = \sum_m E_m e^{i\omega_m t}$$

At twice threshold,

$$N = \frac{\Delta\nu}{\delta\nu} \quad \text{\# lasing modes}$$

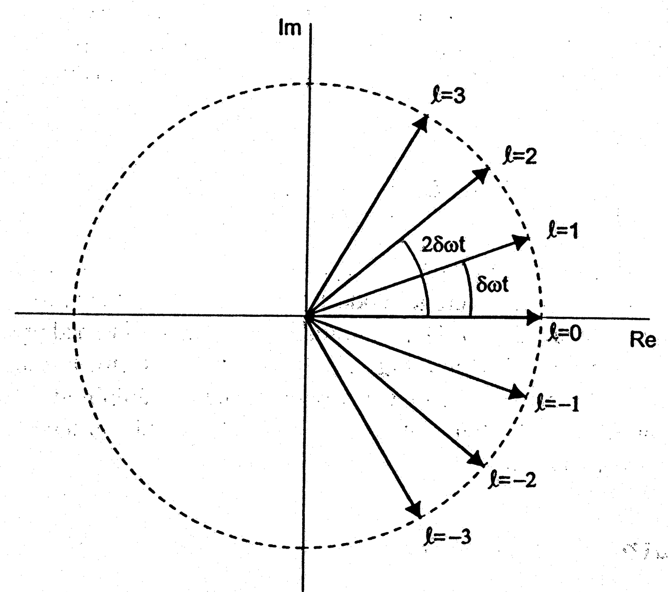
Take
$$E_m = \begin{cases} E_0 & \text{for } N \text{ lasing modes} \\ 0 & \text{all other modes} \end{cases}$$

Let
$$\omega_m = \omega_0 + l \cdot \delta\omega \qquad l = 0, \pm 1, \pm 2, \ldots \pm \frac{(N-1)}{2}$$

Then

$$E(t) = E_0 e^{i\omega_0 t} \sum_{l=-N/2}^{N/2} e^{il(\delta\omega t)}$$

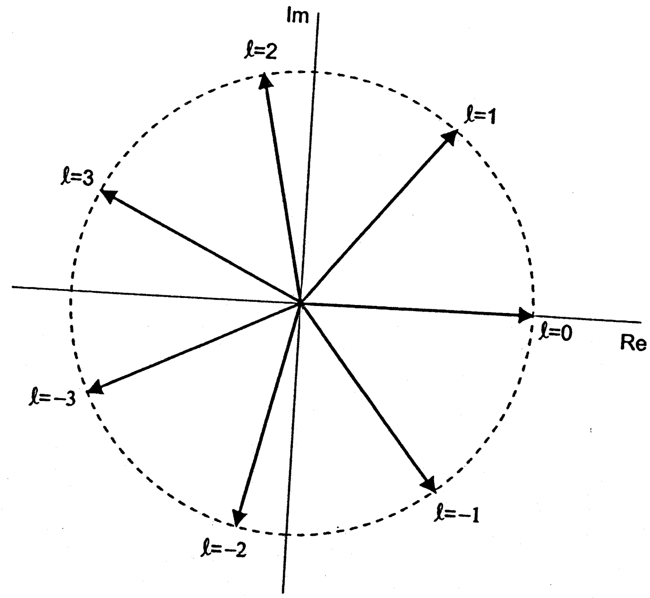
Phasors fan out as $(\delta\omega t)$ increases



At $t = 0$, phasors in line $\Rightarrow E = NE_0$

At $\frac{N}{2}\delta\omega t = \pi$, phasors cancel $\Rightarrow E = 0$

Light pulse has duration



$$\frac{N}{2}\delta\omega\,\Delta t_p = \pi$$

$$\Delta t_p \cong \frac{2\pi}{N\delta\omega} = \frac{1}{N\delta\nu}$$

and using

$$N \cong \frac{\Delta\nu}{\delta\nu}$$

then

$$\boxed{\Delta t_p \cong \frac{1}{\Delta\nu}}$$

* uncertainty relation
* transform limited pulse
* need wide bandwidth for short pulses

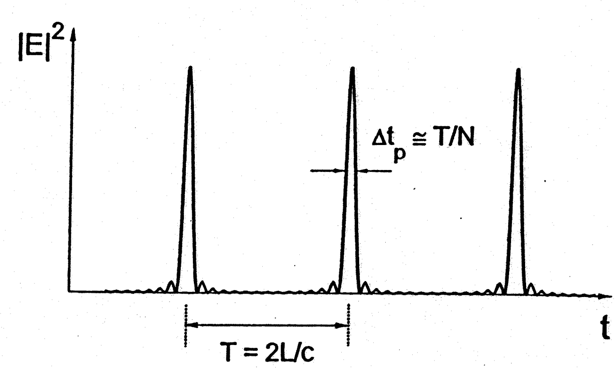
At $\delta\omega t = 2\pi$, all phasors in line again: get another pulse

$$T = \frac{2\pi}{\delta\omega} = \frac{1}{\delta\nu} = \frac{2L}{c}$$ repetition time (also cavity round-trip time)

Pulse width can be written

$$\Delta t_p = \frac{1}{N\delta\nu} = \frac{T}{N}$$

$$\boxed{\frac{\Delta t_p}{T} = \frac{1}{N}}$$



Example: Fiber laser

$L \approx 5$ m

$\lambda = 1530$ nm

$\Delta\lambda = 30$ nm

Find

a) # lasing modes

b) Δt_p

Sol'n: a) $\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda = \frac{3 \cdot 10^8 \text{ m/s}}{1.53 \cdot 10^{-6} \text{ m}} \cdot \left(\frac{30 \text{ nm}}{1530 \text{ nm}}\right)$

$$\Delta\nu = 3.84 \cdot 10^{12} \text{ Hz}$$

$$\delta\nu = \frac{c}{2nL} = \frac{3 \cdot 10^8 \text{ m/s}}{2(1.5)(5)\text{m}} = 2.0 \cdot 10^7 \text{ Hz} \approx 20 \text{ MHz}$$

$$N = \frac{\Delta\nu}{\delta\nu} = \frac{3.84 \cdot 10^{12}}{2 \cdot 10^7} = \boxed{1.92 \cdot 10^5}$$

b) $\Delta t_p = \frac{1}{\Delta\nu} = \frac{1}{3.84 \cdot 10^{12}} = 2.60 \cdot 10^{-13} \text{ s} = \boxed{260 \text{ fs}}$

Energy in mode-locked pulse

* Mode-locking doesn't change average power
  Only re-distributes power in time

* Evaluate total energy in one round-trip time for both cw (non mode-locked) and pulsed:

$$P_{cw} \cdot T = P_{ML} \cdot \Delta t_p = P_{ML} \cdot \frac{T}{N}$$

$$\therefore \quad \boxed{P_{ML} = N P_{cw}}$$

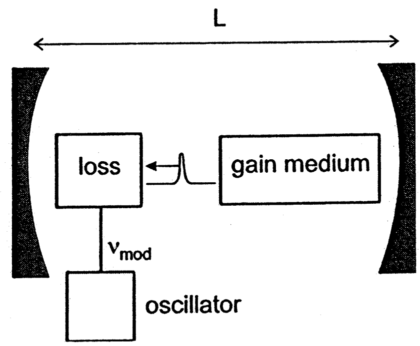
* Peak power greatly enhanced in mode-locking
  Useful for nonlinear optics applications

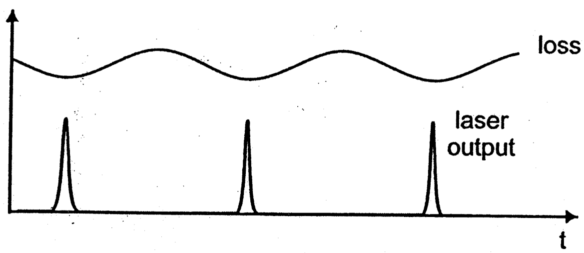
Methods for mode locking

(1) Active mode locking

set $\nu_{mod} = \frac{1}{T}$

"shutter" opens when pulse arrives

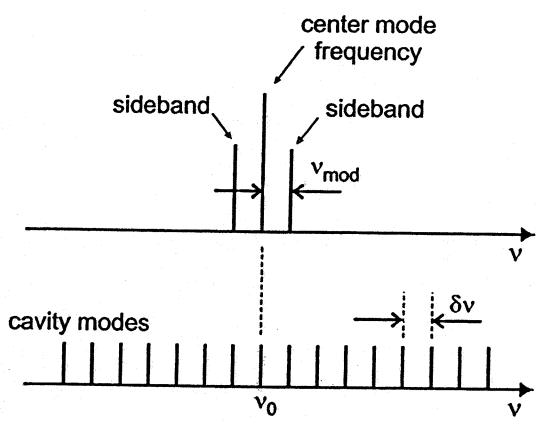




For $L = 1\,m$

$$\delta\nu = \frac{c}{2L} = 1.5 \cdot 10^{8}\ Hz$$
$$= 150\ MHz$$

Need stable RF oscillator

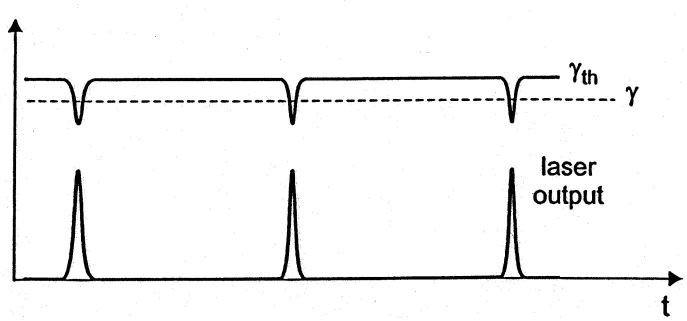


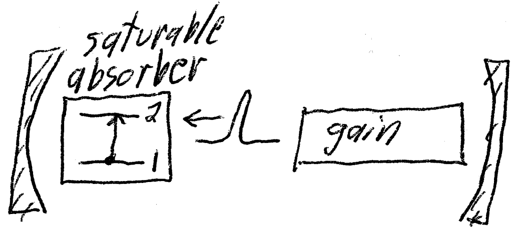
* In frequency domain, modulation creates sidebands that couple modes and lock the phases

(2) Passive mode locking

* similar to Q-switching use saturable absorber

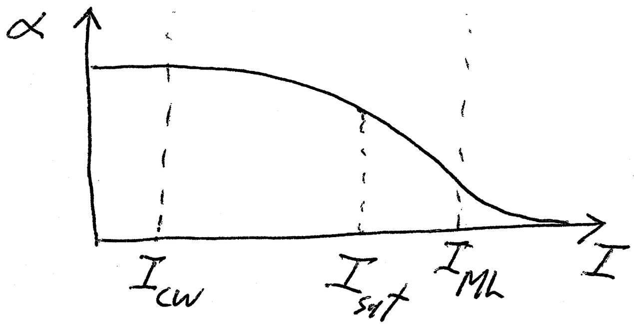
* set $\gamma < \gamma_{th}$ for no mode locking



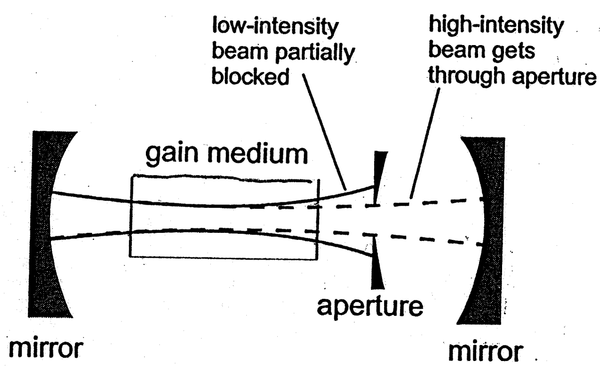


Mode-locked intensity is high enough to saturate absorption and bring $\gamma_{th} < \gamma$

$$I_{ML} = N I_{cw}$$



Alternative is Kerr lens mode locking:



* self-focusing for high intensity pulse
* fast time response due to nonlinearity of electron cloud position

Time scales in pulsed lasers:

1. Pump pulse ~ ms
2. Spiking ~ μs
3. Q-switching ~ ns
4. Mode locking ~ ps