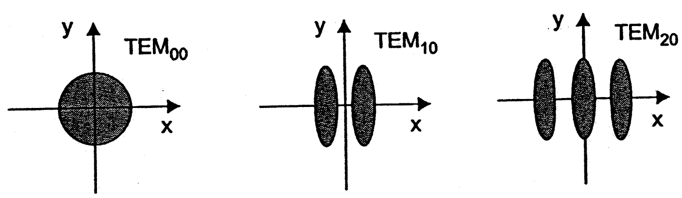
Higher Order Modes

The Gaussian beam is the simplest and lowest order mode of a family of solutions to Maxwell's equations called

Hermite-Gaussian mode $\qquad TEM_{lm} \qquad l = 0, 1, 2, \ldots \quad m = 0, 1, 2, \ldots$

Gaussian beam $\qquad TEM_{00}$



All these can be generated in laser

$$\begin{cases} \text{Transverse modes} & (l, m) \\ \text{Longitudinal modes} & q \qquad \left[\nu = q \frac{c}{2L}\right] \end{cases}$$

* A given "mode" is described by 3 parameters: (q, l, m) similar to (m_1, m_2, m_3) for rectangular cavity.

* Frequency of mode is

$$\nu_{qlm} = \frac{c}{2L}\left\{ q + \frac{1+l+m}{\pi} \cos^{-1}(g_1 g_2) \right\}$$

where $g_{1,2} = 1 - \frac{L}{r_{12}}$

limit as $r \to \infty$

$\nu_{qlm} \to \frac{c}{2L}$

These higher order modes are called "Hermite - Gaussian", with transverse variation

$$E(x,y) = H_\ell\left(\frac{\sqrt{2}}{w}x\right) H_m\left(\frac{\sqrt{2}}{w}y\right) e^{-\frac{(x^2+y^2)}{w^2}}$$

$H_m(u)$ are "Hermite polynomials"

$$H_0(u) = 1$$

$$H_1(u) = u$$

$$H_2(u) = 2u^2 - 1$$

$$H_3(u) = 2u^3 - 3u$$

There are ℓ zero's in x direction
m zero's in y "

$\ell+1$ maxima in x
$m+1$ maxima in y

Can determine w from location of intensity zero's.

Effective width of higher order modes is larger than w by factor M:

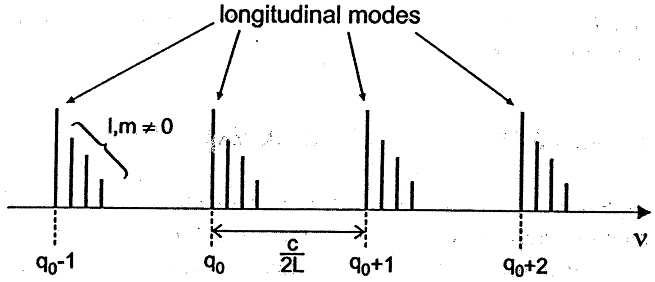
$$\left[\begin{array}{l} w_{eff} \approx M\, w(z) \\ w_{0,eff} \approx M\, w_0 \end{array}\right]$$

but beam still diverges according to

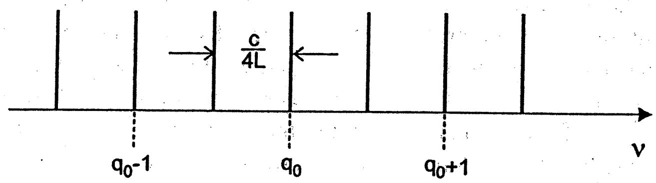
$$w(z) \simeq \frac{\lambda}{\pi w_0} \cdot z$$

so

$$w_{eff} = M \frac{\lambda}{\pi w_0} z = M^2 \frac{\lambda}{\pi w_{0,eff}} z$$



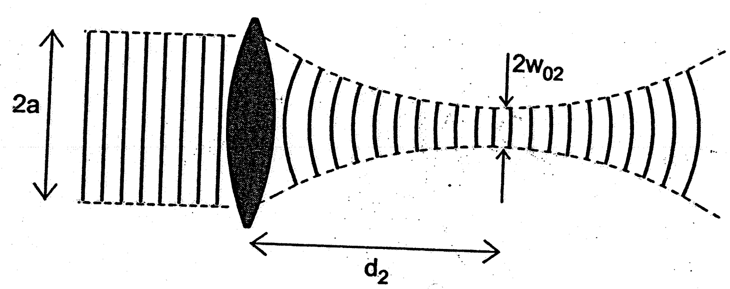
near planar $g \to 1$



confocal $g = 0$

Focusing Gaussia Beams

Important properties of laser light
* focus light to small spot
  high energy density (spatial coherence)

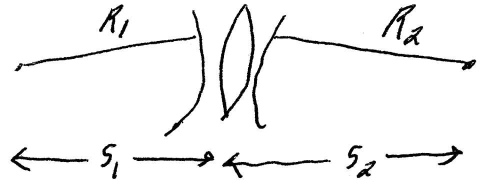


Lens changes one Gaussian beam into another

In general, for thin lens:

(1) W same each side

(2) $\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f}$



sign convention:
R + diverging
R − converging

$R_1 = s_1$
$R_2 = -s_2$

$\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$

Exact method:

Solve (1) $W^2(z) = W_0^2\left[1 + \left(\frac{z}{z_0}\right)^2\right]$

(2) $R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$

$$z_0 = \frac{\pi W_0^2}{\lambda}$$

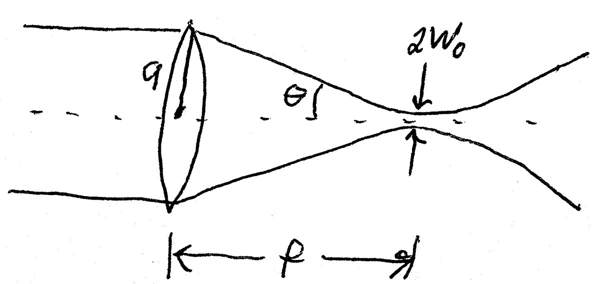
for d_2 and W_0 using $W(-d_2) \approx a$

$R(-d_2) = -f$

Rather messy algebra.

Approximate method:

Beam waist approx at $d_2 \simeq f$



$\theta \simeq \frac{a}{f} \simeq \frac{\lambda}{\pi W_0}$

$\therefore$ $\boxed{W_0 \simeq \frac{\lambda f}{\pi a}}$

To get small spot:

1) short λ
2) short f
3) large a

largest a limited by lens dia $D = 2a_{max}$

$$W_{min} = \frac{2\lambda f}{\pi D} = \frac{2}{\pi}\lambda F^{\#}$$

where $F^{\#} \equiv f/D$ — F-stop on cameras

Smaller $F^{\#}$: tighter focus
more aberrations
more expensive

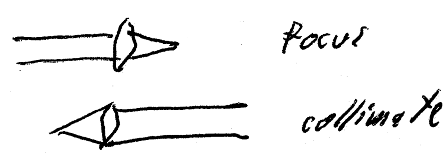
Typical $F^{\#} \sim 4$

$$\left.\begin{array}{ll} \lambda \sim 10^{-6}\ m & \\ f \sim 10^{-1}\ m & \\ a \sim 10^{-3}\ mm & (\text{Ar laser}) \end{array}\right\} \quad W_0 \sim \frac{(10^{-6})(10^{-1})}{\pi(10^{-3})} \sim 3 \cdot 10^{-5}\ m$$

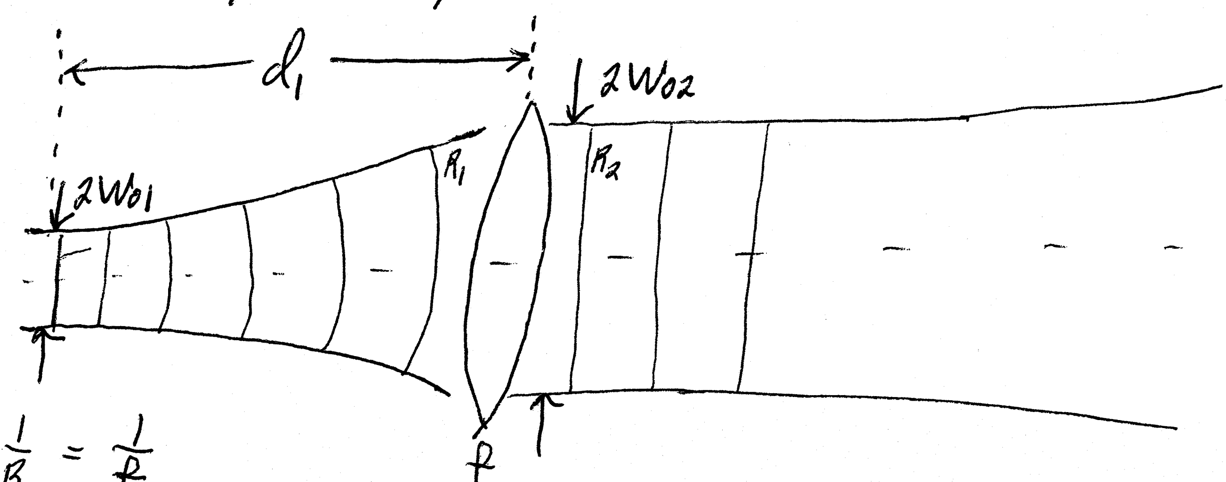
$\sim 30\ \mu m$

Collimating Gaussian Beam

Opposite problem from focusing



What focal length to pick?



$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f}$$