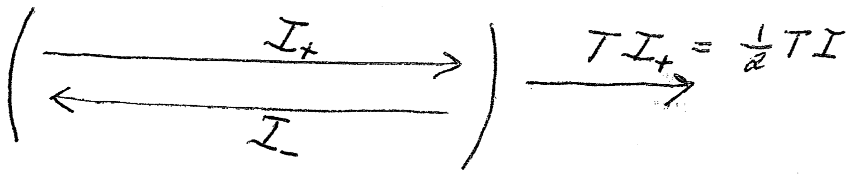
using $P_{in} = R V h\nu_p$

$P_{out} \approx \frac{1}{2} I A T$




where we assume only transmission through 1 mirror.

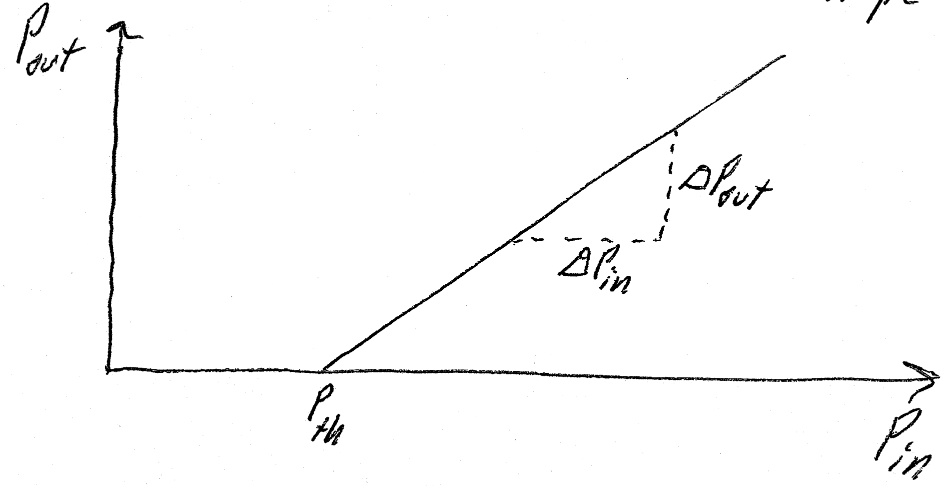
$$P_{out} = \frac{AT}{2} \frac{h\nu}{\sigma\tau_2}\left[\frac{P_{in}}{P_{th}} - 1\right]$$

$$P_{out} = \frac{1}{2} A T I_s \left[\frac{P_{in}}{P_{th}} - 1\right] \qquad \text{since } I_s \equiv \frac{h\nu}{\sigma\tau_2}$$

$$P_{out} = \frac{1}{2} A T \frac{I_s}{P_{th}}\left[P_{in} - P_{th}\right]$$

$$P_{out} = \eta_s \left[P_{in} - P_{th}\right] \qquad \eta_s \equiv \frac{\Delta P_{out}}{\Delta P_{in}}$$

slope efficiency



$$\frac{1}{\tau_c} = \left[\frac{\text{frac loss}}{\text{unit time}}\right] = \underbrace{\left[\frac{\text{distance}}{\text{time}}\right]}_{c}\underbrace{\left[\frac{\text{frac lost}}{\text{unit distance}}\right]}_{\alpha}$$

$$\frac{1}{c\tau_c} = \frac{\text{frac. lost}}{\text{unit distance}}$$

From p. 57,

$$\gamma_{th} \approx \frac{\text{frac loss}}{\text{distance}} = \frac{1}{c\tau_c} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

$$\gamma_{th}\,2L \approx \frac{\text{frac loss}}{RT} \approx 2\alpha L + \ln\left(\frac{1}{R_1R_2}\right)$$

For $R_1R_2 \approx 1$, $\quad \ln\left(\frac{1}{1-T}\right) \approx \ln(1+T) \approx T$

Let $R_1 = 1$

$R_2 \approx 1-T$

$R_1R_2 \approx 1-T$

So $\quad 2L\gamma_{th} \approx \underbrace{2\alpha L}_{\delta} + T$

$$\frac{1}{c\tau_c} = \gamma_{th} = \frac{\delta + T}{2L}$$

For our simple model, the slope efficiency is

$$\eta_s = \frac{1}{2} A T \frac{I_s}{P_{th}}$$

Using $I_s \cong \frac{h\nu}{\sigma \tau_2}$ and $P_{th} \simeq \frac{V h\nu_p}{c \tau_c \sigma \tau_2}$

$$\eta_s = \frac{1}{2} A T \frac{h\nu}{h\nu_p} \frac{c\tau_c}{V} = T \frac{h\nu}{h\nu_p} \frac{c\tau_c}{2L}$$

Recall

$$\frac{\text{frac. loss}}{RT} = \frac{2L}{c\tau_c} = \underbrace{\alpha (2L)}_{\delta} + \underbrace{(1 - R_1 R_2)}_{T}$$

$$\delta = \frac{\text{frac. internal loss}}{RT}$$

$$T \approx \frac{\text{frac. coupling loss}}{RT}$$

assume $R_1 \simeq 1$
$R_2 \simeq 1 - T$

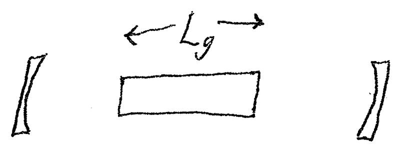
Then $\boxed{\eta_s = \frac{T}{\delta + T} \frac{h\nu}{h\nu_p}}$ limited by quantum defect

Slope efficiency maximized at $T \gg \delta$ where cavity "losses" mostly due to output coupling

But is laser output maximized at $T \gg \delta$ ? Consider threshold.

Example:

Consider again Nd: YAG cavity




$R_1 = 1$ $\qquad$ $R_2 \approx 0.85$

$L_g = 7.5$ cm

$\lambda_p = 500$ nm

$\lambda \approx 1064$ nm

$\alpha = 5 \cdot 10^{-3}$ cm$^{-1}$

$$T = 1 - R_2 = 0.15$$

$$\delta = 2\alpha L_g = 2(5 \cdot 10^{-3}\,\text{cm}^{-1})(7.5\,\text{cm}) = 0.075$$

$$\therefore \quad \eta_s = \left(\frac{0.15}{0.075 + 0.15}\right)\frac{h\nu}{h\nu_p}$$

$$\eta_s \approx 0.667 \frac{h\nu}{h\nu_p}$$

$$\approx 0.667 \left(\frac{hc/\lambda}{hc/\lambda_p}\right) = 0.667\left(\frac{\lambda_p}{\lambda}\right)$$

$$\eta_s = (0.667)\left(\frac{500}{1064}\right) = 0.313$$

no lasing : $N_2 \cong R\tau_2$ $\qquad N_2 \ll N$ $\qquad \boxed{62B}$

$\gamma_0 \approx N_2 \sigma$

$$\gamma_0 \cong R\sigma\tau_2 \approx \frac{P_p}{V h\nu_p}\sigma\tau_2$$

$$\delta + T \approx \frac{2L}{c\tau_c} \propto P_{th}$$

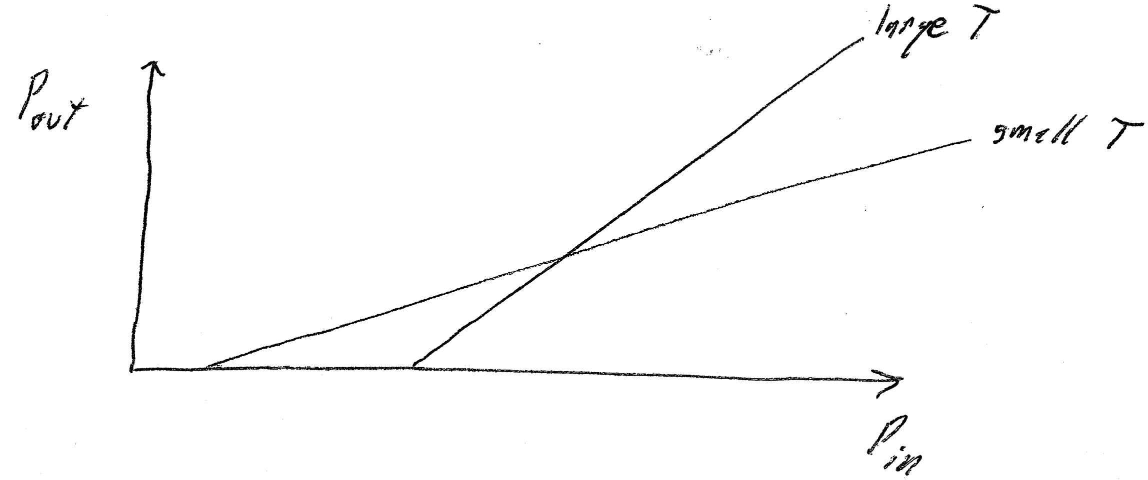
$$\therefore \quad \frac{P_p}{P_{th}} = \frac{\gamma_0 2L}{\delta + T}$$

since at $P_p = P_{th}$, $\quad \gamma_0 2L \approx \delta + T$

$$P_{th} = \frac{V\, h\nu_p}{c\,\tau_c\,\sigma\,\tau_2} = (\delta + T)\left(\frac{V}{2L}\right)\frac{h\nu_p}{\sigma\,\tau_2}$$

$$I_{th} \simeq \frac{1}{2}(\delta + T)\, I_s\, \frac{h\nu_p}{h\nu}$$

This increases with $T$ for $T \gg \delta$




So optimum $T$ depends on pump rate. For higher $P_{in}$, larger $T$ maximizes $P_{out}$.

Rewrite $P_{out}$ vs $P_{in}$ to emphasize relation of gain to loss

$$P_{out} = \frac{1}{2} A T I_s \left[\frac{P_{in}}{P_{th}} - 1\right]$$

Use $\quad P_{in} = R\, V\, h\nu_p \qquad \gamma_0 = R\,\sigma\,\tau_2$

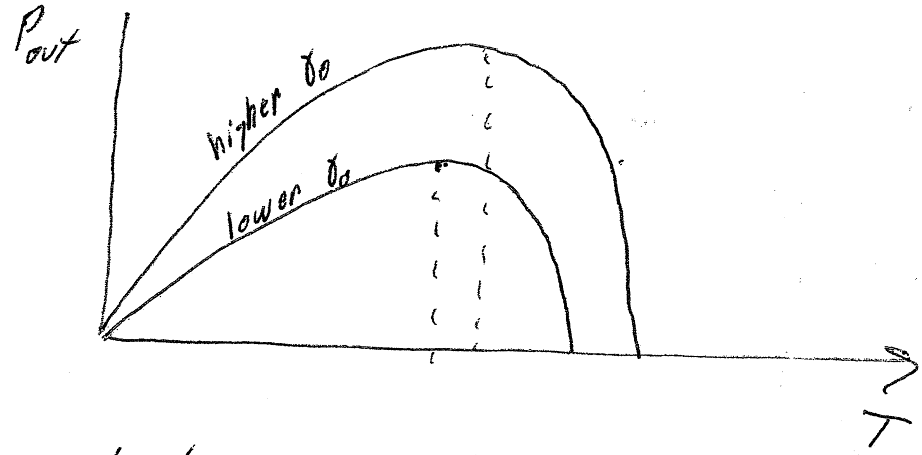
unsaturated gain coeff

$$P_{th} = R_{th}\, V\, h\nu_p \qquad R_{th} = \frac{1}{c\,\tau_c\,\sigma\,\tau_2}$$

$$\therefore \quad \frac{P_{in}}{P_{th}} \simeq \frac{R}{R_{th}} = \frac{\gamma_0}{\sigma\,\tau_2} \cdot c\,\tau_c\,\sigma\,\tau_2$$

$$r \equiv \frac{P_{in}}{P_{th}} = \gamma_0\, c\,\tau_c = \frac{\gamma_0\, 2L}{\delta + T}$$

$$\boxed{P_{out} = \frac{1}{2} A T I_s \left[ \frac{\gamma_0 2L}{\delta + T} - 1 \right]}$$




Optimum output occurs at higher $T$ as unsaturated gain $\gamma_0$ increases.

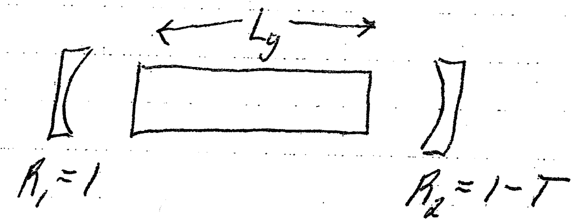
To optimize, set $\frac{dP_{out}}{dT} = 0$

Find $\boxed{T_{opt} = \sqrt{(\gamma_0 2L)\,\delta} - \delta}$

Note: we have made "high Q" approx here, so this only valid for small $T$

Example:

Same Nd:YAG cavity, pumped at twice the threshold rate when $R_2 = 0.85$



$L_g = 7.5$ cm

$\sigma_{peak} = 2.8 \cdot 10^{-19}$ cm$^2$

$\tau_{21} = 230\ \mu s$

$\lambda = 1064$ nm

$\alpha = 5 \cdot 10^{-3}$ cm$^{-1}$

With $R_2 = 0.85$

$$\mathcal{R} = 2\,\mathcal{R}_{th} = 4.9 \cdot 10^{20} \frac{1}{cm^3 \cdot s}$$

Find optimum $T$ for maximum output power.

$$\gamma_0 = \mathcal{R}\sigma\tau_{21} = \left(4.9 \cdot 10^{20} \tfrac{1}{cm^3 \cdot s}\right)\left(2.8 \cdot 10^{-19} cm^2\right)\left(2.3 \cdot 10^{-4} s\right)$$

$\gamma_0 = 0.0315$ cm$^{-1}$    unsaturated gain coeff.

$\delta = 0.075$    round-trip internal loss fraction

$$T_{opt} = \sqrt{(0.0315)(15)(0.075)} - 0.075$$

$$\boxed{T_{opt} = 0.113}$$

So the value $T = 0.15$ is higher than optimum for this pumping power