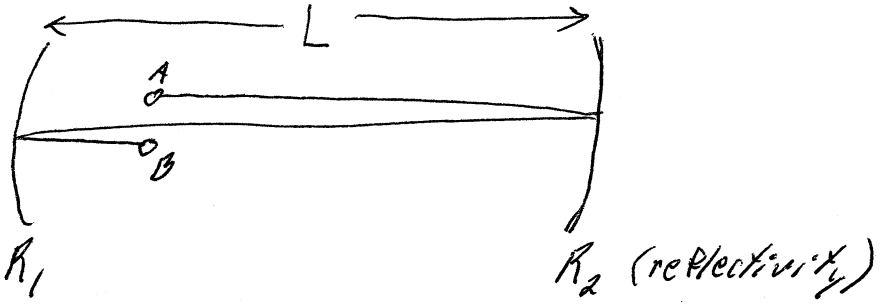
Threshold For Laser Operation



$$I_B = R_1 R_2 e^{(\gamma-\alpha)2L} I_A$$

γ = gain coeff

α = loss coeff (internal)

Lasing occurs when gain is great enough to offset losses so $I_B \geq I_A$

Need $\boxed{R_1 R_2 e^{2(\gamma-\alpha)L} \geq 1}$

Take log:

$$\ln(R_1 R_2) + 2(\gamma-\alpha)L \geq 0$$

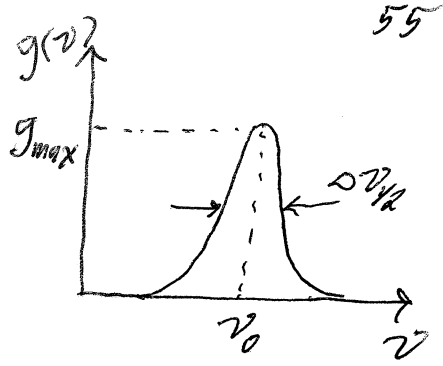
$$\gamma \geq \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right) \equiv \gamma_{th}$$

Using $\gamma(\nu) = A_{21}\frac{\lambda^2}{8\pi} g(\nu)\,\Delta N$

Then $A_{21}\frac{\lambda^2}{8\pi} g(\nu)\,\Delta N \geq \gamma_{th}$

Required ΔN for lasing:

$$\Delta N \geq \frac{8\pi \gamma_{th}}{A_{21} \lambda^2 g(\nu)}$$



Easiest to lase (need less ΔN) at ν_0:

$$\Delta N \geq \frac{8\pi \gamma_{th}}{A_{21} \lambda^2 g_{max}} = \frac{4\pi^2 \cdot \gamma_{th} \Delta\nu_{1/2}}{A_{21} \lambda^2}$$

Lowest threshold for

1) low losses α
2) high reflectivity mirrors
3) longer cavities
4) narrow linewidth $\Delta\nu_{1/2}$
5) large radiative decay rate A_{21}

Example: $\alpha = 0.02\ m^{-1}$ fiber laser

$R_1 \simeq R_2 \simeq 0.99$

$L = 5\ m$

$\Delta\lambda = 50\ nm$ in glass

$\lambda = 1500\ nm$ Er

$\tau_{21} = 10\ ms$ dipole forbidden

$$\Delta\nu = \Delta\left(\frac{c}{\lambda}\right) \simeq \frac{c}{\lambda^2}\Delta\lambda = 6.67 \cdot 10^{12}\ s^{-1}$$

$$\gamma_{th} \simeq \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right) = .02 + .002 = .022$$

$$\therefore\ \Delta N > \frac{4\pi^2(.022\ m^{-1})(6.67\cdot 10^{12}\ s^{-1})(10\cdot 10^{-3}\ s)}{(1.5\cdot 10^{-6}\ m)^2}$$

$$\Delta N > 2.6 \cdot 10^{22}\ m^{-3} \quad \underline{\text{or}} \quad \Delta N > 2.6 \cdot 10^{16}\ cm^{-3}$$

Since in solid there are $\sim 10^{23}$ atoms/cm^3

This requires only $\lesssim 1$ ppm atoms to be excited Er ions.

Typically dope with 1000 ppm, so need 1/1000 excited.

Above Threshold

If $\gamma > \gamma_{th}$ then light intensity grows.

At sufficiently high I, gain saturates