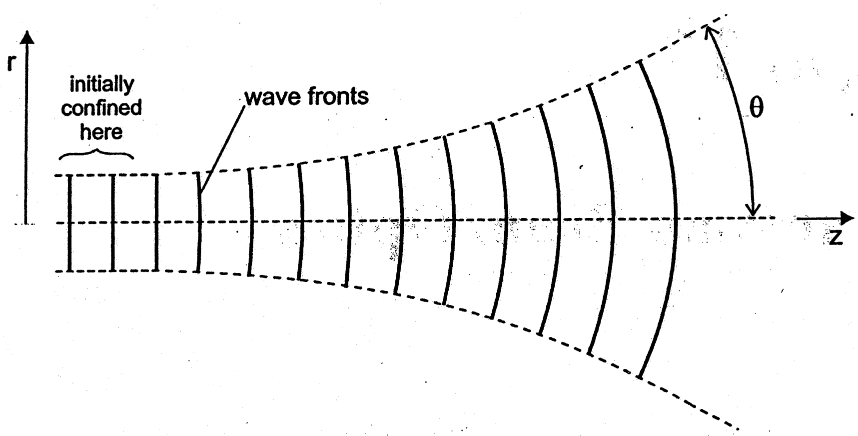
Gaussian Beams

* So far we have considered distribution of light intensity along cavity axis (longitudinal modes)

* Now consider distribution perpendicular to cavity axis (transverse modes)

* 3-D modes characterized by 3 numbers (plus polarization) [Analogy with QM]

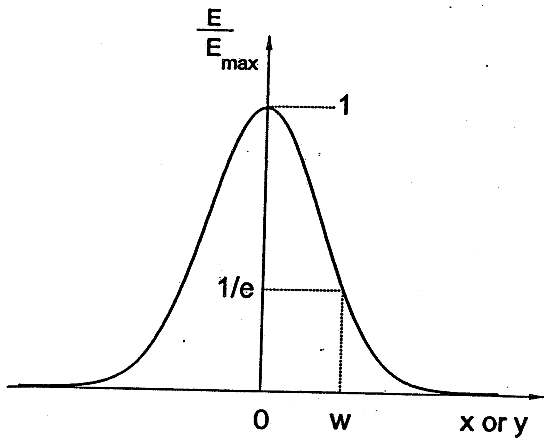
* Many possible modes, each having divergence angle



* The most coherent mode has the lowest divergence angle θ for a given initial beam width

* It is called a Gaussian beam or TEM_{00} mode.

The radial (transverse) distribution follows a Gaussian distribution:



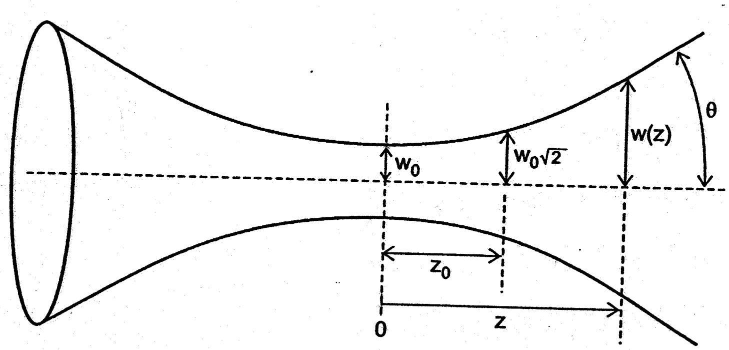
$$E(r,z) = E_0 \frac{w_0}{w(z)} e^{-\frac{r^2}{w^2(z)}}$$

$$r = \sqrt{x^2+y^2}$$

w = beam radius or "spot size"

The spot size w and wavefront radius of curvature R vary with longitudinal position z as

$$\boxed{\begin{aligned} w^2(z) &= w_0^2\left[1+\left(\frac{z}{z_0}\right)^2\right] \\ R(z) &= z\left[1+\left(\frac{z_0}{z}\right)^2\right] \\ z_0 &= \frac{\pi w_0^2}{\lambda} \end{aligned}}$$



w_0 is "beam waist"

size of w_0 and location of beam waist specify everything about a Gaussian beam (for given wavelength)

z_0 is "Rayleigh range"

In limit $z \gg z_0$,

$$W(z) \simeq \frac{W_0}{z_0} \cdot z$$

$$\theta = \frac{W}{z} \cong \frac{W_0}{z_0} = \boxed{\frac{\lambda}{\pi W_0}}$$ half-angle divergence

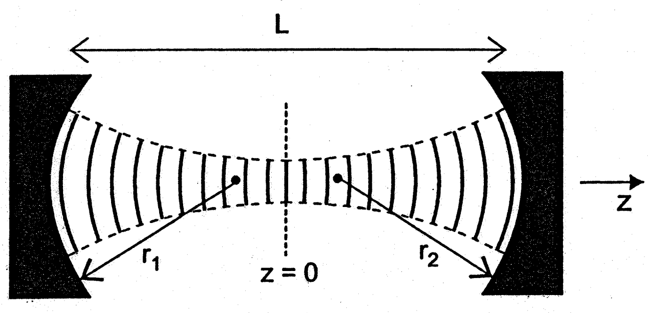
Compare to previous $\theta \sim \frac{\lambda}{D}$

Limits for $R(z)$: $R(0) \to \infty$, $R(\infty) \to \infty$

R is minimum at $z \sim z_0$

Gaussian Beams in Laser Cavities

Which Gaussian beam is produced in laser cavity? The one that "fits"!



r_1 and r_2 are radii of curvature

$r > 0$ concave

$r < 0$ convex

Assume $r_1 = r_2 \equiv r$ Symmetrical cavity

Beam waist at center

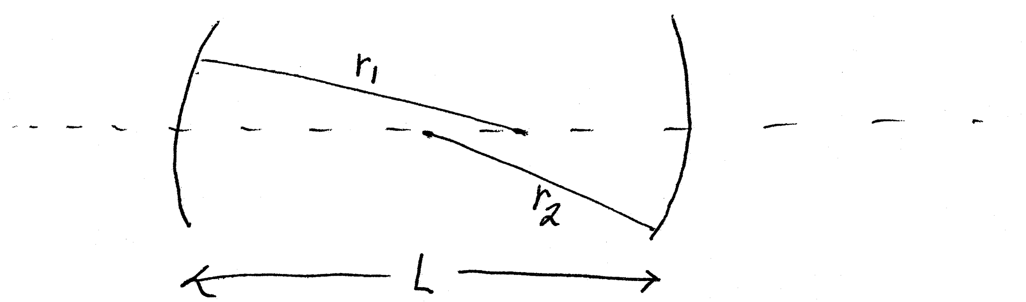
Require $R(\frac{L}{2}) = r$

$$R(\tfrac{L}{2}) = \frac{L}{2}\left[1 + \left(\frac{z_0}{L/2}\right)^2\right] = r$$

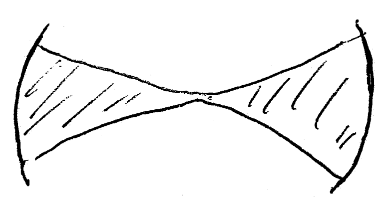
$$1 + \frac{4 z_0^2}{L^2} = \frac{2r}{L}$$

$$z_0 = \frac{L}{2}\sqrt{\frac{2r}{L} - 1}$$

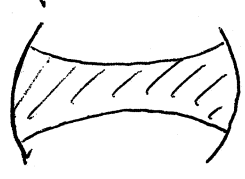
Note that z_0 is real only for $2r \geq L$



$\boxed{2r = L}$ concentric



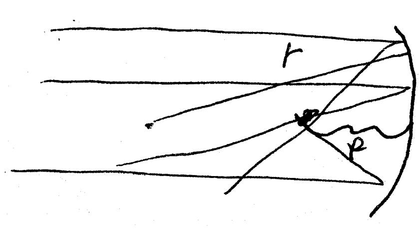
$\boxed{r = L}$ confocal



- good stability
- beam overlaps well with gain medium

focal length of mirror

$$f = \frac{r}{2}$$



Waist size

$$z_0 = \frac{\pi w_0^2}{\lambda} = \frac{L}{2}\sqrt{\frac{2r}{L} - 1}$$

$$\boxed{w_0^2 = \frac{\lambda L}{2\pi}\sqrt{\frac{2r}{L} - 1}}$$

For confocal cavity, $w_0^2 = \frac{\lambda L}{2\pi}$

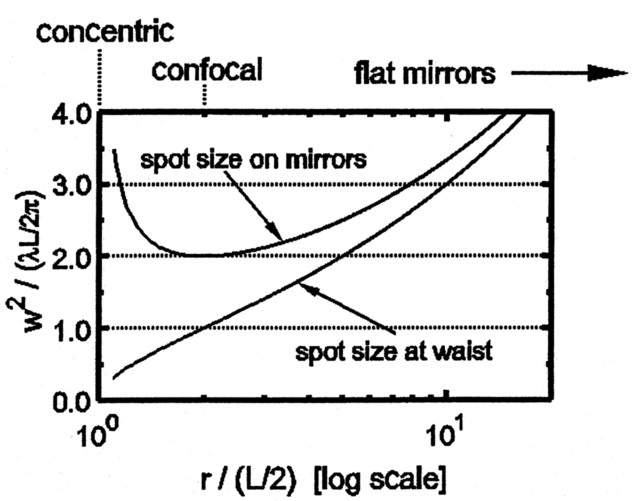
For concentric cavity, $w_0 \to 0$

Spot size on mirrors

$$w^2\left(\tfrac{L}{2}\right) = w_0^2\left[1 + \left(\frac{L/2}{z_0}\right)^2\right]$$

$$= w_0^2\left[1 + \frac{1}{\frac{2r}{L} - 1}\right]$$

$$\boxed{w^2\left(\tfrac{L}{2}\right) = \frac{\lambda L}{2\pi}\,\frac{2r/L}{\sqrt{\frac{2r}{L} - 1}}}$$



General stability criteria:

$$0 < g_1 g_2 < 1$$

$$g_1 \equiv 1 - L/r_1$$

$$g_2 \equiv 1 - L/r_2$$